For full credit show all of your work (legibly!), unless otherwise specified. Answers need not (and probably should not) be completely reduced unless otherwise stated, and may be left in terms of sums, differences, products, quotients, exponentials, factorials, and binomial coefficients.

1. **(15 points)** Computationally, a vector is simply a list of numbers. We may represent an n-dimensional vector $\vec{a}$ as a list of n coordinates $(a_1, a_2, a_3, \ldots, a_n)$.

   (a) **(10 points)** Write an algorithm, using only simple computational steps, to compute the dot product of the vectors $\vec{a}$ and $\vec{b}$. Recall that a dot product of two vectors is the sum of the coordinatewise products, e.g. $(5, 3, 1, -2) \cdot (-1, 0, 4, 3) = 5 \cdot -1 + 3 \cdot 0 + 1 \cdot 4 + (-2) \cdot 3 = -7$.

   Input: sequences $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$
   Output: number c which is equal to $\vec{a} \cdot \vec{b}$

   (b) **(5 points)** Justify and state a good asymptotic bound in big-O notation on the number of steps taken by your algorithm.

2. **(10 points)** Find the closed form of the recurrence relation given by initial conditions $a_0 = 5$, $a_1 = 0$, and $a_n = 2a_{n-1} + 24a_{n-2}$ for $n \geq 2$.

1	/15
2	/10
3	/15
4	/10
5	/20
Σ	/70

3. **(15 points)** Anna, Béla, Charles, Diane, and Edgar have dug up a treasure chest full of identical gold coins which they will share among themselves. According to their particular piratical code, Anna is to be given at least 10 coins, Béla and Charles are each to get either 5 or 6 coins (they could each receive the same or different numbers), Diane may receive any number of coins, and Edgar must receive at least 4 coins. Let a_n represent the number of ways in which n coins might be distributed.

   (a) **(5 points)** Find a formula for the ordinary generating function $\sum_{n=0}^{\infty} a_n z^n$.

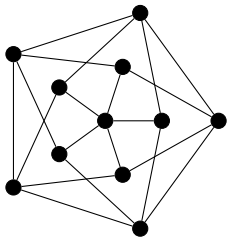
   (b) **(15 points)** Either using the ordinary generating function or by other means, determine how many ways there are to share a chest of 30 coins. You need not arithmetically simplify your answer.

4. **(10 points)** Find the following generating functions:

   (a) **(5 points)** Let a_n be the number of ways to place n *distinct* objects in 5 boxes so that each box contains fewer than 4 items. Determine the formula for the exponential generating function $\sum_{n=0}^{\infty} a_n \frac{z^n}{n!}$.

   (b) **(5 points)** Let b_n be the number of ways to partition n into a sum of the numbers 1, 3, and 4. Determine the formula for the ordinary generating function $\sum_{n=0}^{\infty} b_n z^n$.

5. **(20 points+5 point bonus)** Let G be the graph shown below; label vertices as necessary.



(a) **(10 points)** Demonstrate via an explicit coloring that $\chi(G) \leq 4$, and give an argument that $\chi(G) > 2$.

(b) **(5 points)** Is this graph Eulerian? Explain why or why not.

(c) **(5 points)** Demonstrate that this graph has a subgraph isomorphic to C_6.

(d) **(5 point bonus)** Is this graph planar? Either give an explicit planar representation or explain your reasoning.