

For full credit show all of your work (legibly!), unless otherwise specified. Answers may include all arithmetic operations, trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic simplification of answers is unnecessary.

1. **(13 points)** The *conchoid of de Sluze* is a curve satisfying the equation  $(x-1)(x^2+y^2) = 4x^2$ .

(a) **(10 points)** Find a formula for  $\frac{dy}{dx}$  on this curve.

(b) **(3 points)** Find the equation of the tangent line to the curve at  $(3, -3)$ .

2. **(10 points)** Find  $\frac{d}{dx} \frac{\arcsin x}{\sin e^x - 7}$ .

1	/ 13
2	/ 10
3	/ 19
4	/ 10
5	/ 8
6	/ 10
7	/ 15
8	/ 15
9	/ (6)
$\Sigma$	/100

3. **(19 points)** Answer the following derivative-related questions.

(a) **(6 points)** If  $y = e^{\sqrt{\tan \theta}}$ , find  $\frac{dy}{d\theta}$ .

(b) **(6 points)** Compute  $\frac{d}{dt} \frac{t^3 - \csc t}{\arctan t}$ .

(c) **(7 points)** If  $f(x) = e^{4x} \ln x$ , find  $f'(x)$ .

4. **(10 points)** Find an equation of the tangent line to the curve  $y = \frac{x^2 - 3 \ln x}{x - 2}$  at  $(1, -1)$ .

5. **(12 points)** Milk for homemade yogurt is raised to a temperature of  $180^{\circ}\text{F}$  and then placed into a  $40^{\circ}\text{F}$  ice-water bath to cool. A temperature measurement taken five minutes after the pot is submerged in the ice-water indicates that the milk has cooled to  $145^{\circ}\text{F}$ .

(a) **(5 points)** Produce a function  $f(t)$  modeling the temperature of the milk  $t$  minutes after the pot is put into the ice-water bath.

(b) **(3 points)** How quickly is the milk cooling 2 minutes after it is put into the ice water?

(c) **(4 points)** A lactobacillus culture is to be added to the milk once it has reached  $110^{\circ}\text{F}$  for optimal growth. How long should the milk be chilled before the culture is added?

6. **(8 points)** Estimate the following values using appropriate linear approximations.

(a) **(4 points)**  $\sqrt[3]{1000.3}$

(b) **(4 points)**  $(-2.994)^4$

7. **(10 points)** Calculate  $\frac{d}{dt} [(t^3 + 5t^4) \tan(\ln t)]$ .

8. **(15 points)** A parachuter, currently at a height of 0.4 miles above the ground, is falling straight downwards at a speed of 10 miles per hour. You are 0.3 miles away from the landing site, standing still and recording the descent with a camera.

(a) **(9 points)** How quickly are you and the parachuter approaching each other?

(b) **(6 points)** How quickly should you be tilting the camera in order to keep the parachuter in the frame?

9. **(6 point bonus)** Complete these two questions on the back of this page.

(a) **(3 point bonus)** Prove the quotient rule using any combination of the other rules (the product rule, chain rule, and derivatives of known functions).

(b) **(3 point bonus)** Prove the product rule using only the chain rule and the derivatives of known functions.