No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(17 points)** Answer the following questions:

   (a) **(8 points)** Find the general antiderivative of $g(x) = \frac{x^3+3x+2}{x^2} - \frac{5}{\sqrt{1-x^2}} + \csc x \cot x$.

   (b) **(9 points)** Given that $f(2) = 0$ and $f'(t) = t^3 - \frac{16}{t^3}$, find a formula for $f(t)$.

2. **(24 points)** An industrial manufacturer has budgeted 8000 square feet of floor space for a rectangular factory with a loading dock running along one side. They want to fill it as full as possible with machines, but regulations require 10 feet of open floor space along each wall, and 15 feet of open floor space along the loading dock. What dimensions for the factory will maximize the quantity of floor space available for their machines?

| | |
|---|---|
| 1 | / 17 |
| 2 | / 24 |
| 3 | / 24 |
| 4 | / 12 |
| 5 | / 23 |
| 6 | / (6) |
| $\Sigma$ | /100 |

3. **(24 points)** Answer the following questions related to the shape of the graph of the function $f(x) = x^3 - 6x^2 - 12x + 5$.

   (a) **(4 points)** What are $f(x)$'s long term behaviors as $x$ grows very large and as $x$ grows very negative? Describe each direction in either words or symbols.

   (b) **(6 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.

   (c) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (d) **(8 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

4. **(12 points)** Answer the following questions about approximation with Newton's method:

   (a) **(6 points)** Starting with an initial value of 3, use two iterations of Newton's method to approximate a zero of $f(x) = x^3 - 4x^2 - 2x + 14$. Your answer need not be arithmetically simplified.

   (b) **(6 points)** Choose $x_1 = 4$ to be an initial approximation of $\sqrt{13}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop $x_2$, a better rational approximation of $\sqrt{13}$; also give an arithmetic expression (which need not **and probably should not** be simplified) for the better approximation $x_3$ arising from a second step of Newton's method.

5. **(23 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) $\lim_{\theta\to 0} \frac{\cos\theta}{e^{\theta}+2}$.

   (b) $\lim_{q\to -\infty} qe^{q}$.

   (c) $\lim_{t\to +\infty} \frac{e^{t}}{t^{2}\ln t}$.

   (d) $\lim_{u\to 0} \frac{\sin u - u}{u^{3}}$.

   (e) $\lim_{x\to 0} \frac{x-\sqrt{x}}{e^{x}-1}$.

6. **(6 point bonus)** Prove that if $f(x)$ is a function which is continuous with a continuous derivative and $k$ local minima, it must have between $k-1$ and $k+1$ local maxima.