

3. **(15 points)** The *lemniscate* is a curve satisfying the equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$.

(a) **(10 points)** Find a formula for $\frac{dy}{dx}$ on this curve in terms of x and y .

(b) **(5 points)** Find the equation of the tangent line to the lemniscate at the point $(3, 1)$.

4. **(18 points)** Evaluate the following integrals:

(a) **(6 points)** $\int_0^4 x\sqrt{16 - x^2} dx$.

(b) **(6 points)** $\int \frac{x^3}{x^4+1} dx$.

(c) **(6 points)** $\int_0^{\pi/3} \cos \theta + 2 \sec \theta \tan \theta d\theta$.

5. **(16 points)** A ten-foot-long ladder is leaning against a wall, with the base of the ladder six feet from the wall. The base is sliding away from the wall at a rate of half a foot per hour.

(a) **(8 points)** How quickly is the top of the ladder slipping down the wall?

(b) **(8 points)** How quickly is the angle between the ladder and the floor changing?

6. **(16 points)** Compute the following expressions:

(a) **(6 points)** Compute $\frac{d}{dx} \sqrt{\arctan \sqrt{x}}$.

(b) **(6 points)** Given $f(t) = \tan \frac{e^t}{\arcsin t}$, find $f'(t)$.

(c) **(4 points)** Find $\int \frac{d}{ds} \frac{s^3}{\sqrt{s^2+5}} ds$.

7. **(12 points)** Consider the function $g(x) = \frac{x}{x^2+9}$.

(a) **(5 points)** Identify the zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which, and if any features are not present, say so.

(b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.

(c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

8. **(15 points)** Determine the following limits.

(a) **(5 points)** Evaluate $\lim_{\theta \rightarrow 0} \frac{\theta - \sin \theta}{\theta - \tan \theta}$ or demonstrate that it cannot be evaluated.

(b) **(5 points)** Evaluate $\lim_{t \rightarrow 0} \frac{e^t - 1 - t}{t^2}$ or demonstrate that it cannot be evaluated.

(c) **(5 points)** *Using the difference quotient*, find the derivative with respect to x of the function $f(x) = 20 + 3x - 5x^2$. You may not use L'Hôpital's rule for this problem.

9. **(15 points)** Let $f(x) = 2 + 2x^2 - x^4$.
- (a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.
- (b) **(3 points)** What are the critical points of $f(x)$? Is each a local maximum, a local minimum, or neither?
- (c) **(7 points)** Determine where $f(x)$ is concave up and where it is concave down, and identify points of inflection.
10. **(8 points)** Answer the following questions about the function $f(x) = \frac{x^{2/3}}{x+1}$.
- (a) **(4 points)** What is the domain of $f(x)$?
- (b) **(4 points)** Where does the derivative of $f(x)$ exist?
11. **(6 point bonus)** Find, with explanation, on the back of this page, a general form for the following expressions, for positive integer n :
- (a) **(3 points)** $\frac{d^n}{dx^n}(x^2 e^x)$.
- (b) **(3 points)** $\lim_{x \rightarrow \infty} \frac{x^n}{e^{0.01x}}$.