

For full credit show all of your work (legibly!), unless otherwise specified. Answers may include all arithmetic operations, trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic simplification of answers is unnecessary.

1. **(20 points)** Answer the following derivative-related questions.

(a) **(8 points)** Calculate $\frac{d}{dx} \sin(\ln(\arctan x))$.

(b) **(6 points)** If $f(q) = \frac{\ln q}{q^4}$, find $f'(q)$.

(c) **(6 points)** If $y = \arcsin(3x) \tan x$, find $\frac{dy}{dx}$.

2. **(10 points)** If $y = \csc \frac{t^3 - 3t}{e^t + \sin t}$, find $\frac{dy}{dt}$.

3. **(10 points)** Calculate $\frac{d}{ds}(\cos(s)\tan(s^2 - 3s))$.
4. **(12 points)** A pure 150-gram sample of the radioactive material Cobalt-Thorium-G is taken to a shielded laboratory for testing. 6 days later it is found that, due to radioactive decay, only 125 grams of CoTh-G remain.
- (a) **(5 points)** Produce a function $f(t)$ modeling the quantity of Cobalt-Thorium-G left in the sample after t days.
- (b) **(4 points)** How many days will it take the sample to decay to half of its original mass?
- (c) **(3 points)** How quickly, in grams per day, is the sample decaying 3 days into the experiment?

5. **(15 points)** The *cissoïd of Diocles* is a curve satisfying the equation $x(x^2 + y^2) = 4y^2$.

(a) **(12 points)** Find a formula for $\frac{dy}{dx}$ on this curve.

(b) **(3 points)** Find the equation of the tangent line to the curve at $(2, -2)$.

6. **(8 points)** Estimate the following values using appropriate linear approximations.

(a) **(4 points)** 1.03^5 .

(b) **(4 points)** $\sqrt{99.8}$.

7. **(15 points)** Amy is standing motionless 50 meters east of a north-south road with a radar gun, while Bob, who is 120 meters to the north, is driving south. The radar gun reports how quickly the *distance between Amy and Bob* is changing (which may not be Bob's actual speed).
- (a) **(11 points)** If Bob is driving south at 30 meters per second, what will the radar report as the rate of change of the distance between Bob and Amy?

- (b) **(4 points)** Conversely, if the radar reported a change-rate of 25 meters per second, what would Bob's actual speed be?

8. **(10 points)** Find an equation of the tangent line to the curve $y = e^x(x^2 - 4x + 3)$ at $(0, 3)$.