

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(12 points)** Consider the function $g(x) = \frac{e^x}{x-1}$.
- (a) **(5 points)** Identify the zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which, and if any features are not present, say so.
- (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.
- (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.
2. **(20 points)** We have a rectangular sheet of cardboard which is 3 feet by 8 feet in dimensions. By cutting identical squares out of the corners and folding up the resulting sheet, we can build a box without a lid. What size on our corner cut maximizes the volume of the box so generated?

1	/ 15
2	/ 20
3	/ 12
4	/ 15
5	/ 15
6	/ 15
7	/ 16
8	/ 18
9	/ 8
10	/ 16
11	/ (6)
Σ	/150

3. **(16 points)** Calculate the following derivatives:

(a) **(4 points)** Find $\frac{d}{dt} \int \frac{e^t}{t} dt$.

(b) **(6 points)** Find $\frac{d}{dx} \arctan \frac{x^2-1}{x+2}$.

(c) **(6 points)** Given $g(s) = e^s \cot(s^2)$, find $g'(s)$.

4. **(15 points)** The *keratoid cusp* is a curve satisfying the equation $y^2 = x^2y + x^5$.

(a) **(10 points)** Find a formula for $\frac{dy}{dx}$ on this curve in terms of x and y .

(b) **(5 points)** Find the equation of the tangent line to the keratoid cusp at the point $(2, -4)$.

5. **(18 points)** Evaluate the following integrals:

(a) **(6 points)** $\int_{-1}^2 (x^2 - 4)dx.$

(b) **(6 points)** $\int_0^2 \frac{2x}{(x^2+1)^2} dx.$

(c) **(6 points)** $\int (t^5 - t)\sqrt{t^6 - 3t^2} dt.$

6. **(16 points)** A sentry at Blackgate Prison has turned a spotlight on an escapee who is currently 0.3 miles to the north and 0.4 miles to the east of the prison. She notices that the escapee is traveling eastwards at three miles per hour.

(a) **(8 points)** How quickly will she need to rotate the spotlight to keep it trained on the escapee?

(b) **(8 points)** How quickly is the escapee's distance from the prison changing?

7. **(15 points)** Determine the following limits.

(a) **(5 points)** Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6}-2}{x-2}$ or demonstrate that it cannot be evaluated.

(b) **(5 points)** Evaluate $\lim_{x \rightarrow \infty} \frac{x^3+x \ln x}{3x^2-5}$ or demonstrate that it cannot be evaluated.

(c) **(5 points)** *Using the difference quotient*, find the derivative with respect to x of $f(x) = 3x^2 - 4x + 2$. You may not use L'Hôpital's rule for this problem.

8. **(15 points)** Let $f(x) = 2x^3 + 3x^2 - 36x$.

(a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.

(b) **(3 points)** What are the critical points of $f(x)$? Is each a local maximum, a local minimum, or neither?

(c) **(7 points)** Determine where $f(x)$ is concave up and where it is concave down, and identify points of inflection.

9. **(8 points)** Answer the following questions about the function $h(x) = \sqrt{25 - x^2}$.
- (a) **(4 points)** What is the domain of $h(x)$?
- (b) **(4 points)** Where does the derivative of $h(x)$ exist?
10. **(15 points)** Miranda has just taken a 40mg intravenous dose of G-23 paxilon hydrochlorate. In two hours the level of the drug in her system, which is subject to exponential decay, will have reduced to 25mg.
- (a) **(5 points)** Construct a function modeling the quantity of the drug in her body after t hours.
- (b) **(6 points)** How quickly is the drug being eliminated after 2 hours?
- (c) **(4 points)** The experimental protocol requires that subjects be kept under careful observation until the level of the drug in their body is below 10mg. After how many hours can Miranda be released from observation?