1. **(7 points)** *Identify the domains of the following functions:*

   (a) **(3 points)** $h(t) = \ln(25 - x^2)$.
   Since the parameter of the logarithms function must be positive, this function is evaluatable only when $25 - x^2 > 0$; or in other words, when $x^2 < 25$. This is true when $|x| < 5$, which is to say, $-5 < x < 5$, or in interval notation, $(-5, 5)$.

   (b) **(4 points)** $f(x) = \frac{\sqrt{4-x}}{x^2-8x}$.
   This function has two potential crises: the fraction must have a nonzero denominator, and the square root must have a non-negative argument. Thus, we require that $4 - x \geq 0$ and that $x^2 - 8x \neq 0$. The former prohibition can be written as $x \leq 4$; the latter, on solving the quadratic, becomes $x \neq 0, 8$. Thus, our domain consists of all $x \leq 4$ except for 0 (and 8, but that's larger than 4). Written in interval notation, this domain would be $(-\infty, 0) \cup (0, 4]$.

2. **(6 points)** *Below, let* $f(x) = 2x^2 - 3x + 1$ *and* $g(x) = 8x^3 + 1$.

   (a) **(2 points)** *Find formulas for* $f(g(x))$ *and* $g(f(x))$. *You do not need to simplify.*

$$f(g(x)) = f(8x^3 + 1) = 2(8x^3 + 1)^2 - 3(8x^3 + 1) + 1$$
$$g(f(x)) = g(2x^2 - 3x + 1) = 8(2x^2 - 3x + 1)^3 + 1$$

   (b) **(4 points)** *Find a formula for* $g^{-1}(x)$.
   We want $g^{-1}(x)$ to be a function such that $g(g^{-1}(x)) = x$. Thus:

$$\begin{aligned} g(g^{-1}(x)) &= x \\ 8\left[g^{-1}(x)\right]^3 + 1 &= x \\ 8\left[g^{-1}(x)\right]^3 &= x - 1 \\ \left[g^{-1}(x)\right]^3 &= \frac{x-1}{8} \\ g^{-1}(x) &= \sqrt[3]{\frac{x-1}{8}} \end{aligned}$$

3. **(7 points)** *This is the record of the first 5 seconds of a runner's performance in a race:*

Time elapsed (in seconds)	*0.00*	*1.00*	*2.00*	*3.00*	*4.00*	*5.00*
Distance traveled (in meters)	*0.00*	*1.00*	*5.00*	*8.50*	*13.00*	*17.00*

   (a) **(2 points)** *What is the runner's average speed in the first two seconds of the race?*
   Between times $t = 0$ and $t = 2$, the runner proceeds from position 0m to position 5m. Since the runner has traveled 5 meters in 2 seconds, her average speed is $\frac{5}{2} = 2.5$ meters per second.

   (b) **(2 points)** *What is the runner's average speed between the times* $t = 1$ *and* $t = 4$*?*
   Between times $t = 1$ and $t = 4$ (which are temporally separated by a time of three seconds), the runner travels from position 1m to position 13m, traveling a total of 12 meters. Since the runner has progressed 12 meters in 3 seconds, her average speed over this time period is $\frac{12}{3} = 4$ meters per second.

(c) **(3 points)** *The detailed records indicate that 2.99 seconds into the race, they had progressed 8.46 meters. Based on this information, what would be a good estimate for the instantaneous speed after 3 seconds?*

We might argue that a hundredth of a second is very nearly an instant, and thus that the average speed between times $t = 2.99$ and $t = 3.00$ is "close enough" to the concept of instantaneous speed for our purposes. Thus, we may note that in this 0.01-second interval, the runner travels a distance of 0.04 meters (since she moves from position 8.46 to 8.50), and thus that her average speed (which approximates the instantaneous speed quite well) is $\frac{0.04}{0.01} = 4$ meters per second.