1. **(4 points)** *Find a value for the parameter p such that the function* $f(x) = \begin{cases} 2^x & \text{if } x \le 3 \\ px + 4 & \text{if } x > 3 \end{cases}$ *is continuous everywhere.*

The functions given by $y = 2^x$ and $y = px + 4$ are both fairly tame expressions which are continuous everywhere; thus the only issue is making sure that at the point where one switches to the other we also achieve continuity. Specifically, we will need it to be the case that

$$\lim_{x \to 3-} f(x) = \lim_{x \to 3+} f(x) = f(3)$$

These two limits evaluate to 8 and $3p + 4$ respectively; $f(3) = 8$ as well so we need simply ensure that $3p + 4 = 8$, which is true when $p = \frac{4}{3}$.

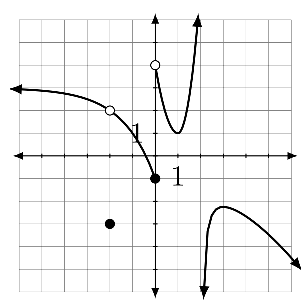
2. **(5 points)** *Prove that* $\lim_{t \to 6} -5 - 2t = -17$ *using epsilon-delta methods.*

The statement $\lim_{t \to 6} -5 - 2t = -17$ is an assertion that, for every value $\epsilon > 0$, a value δ can be furnished such that, if $0 < |t - 6| < \delta$, then $|-5 - 2t - (-17)| < \epsilon$. We may justify this assertion by explicity determining how δ is calculated from ϵ to make this inference true.

$$\begin{aligned} |-5 - 2t - (-17)| &< \epsilon \\ |12 - 2t| &< \epsilon \\ |-2(t - 6)| &< \epsilon \\ |-2| \cdot |t - 6| &< \epsilon \\ 2|t - 6| &< \epsilon \\ |t - 6| &< \frac{\epsilon}{2} \end{aligned}$$

so we may declare that the choice of δ equal to $\frac{\epsilon}{2}$ is sufficient to meet whatever challenge we are given.

3. **(6 points)** *Below is the graph of a function* $f(x)$*. For each of the six quantities listed, give its value if it has a value, or specifically state that it does not exist.*



$\lim_{x \to 2^-} f(x)$ does not exist, because as x approaches 2 fronm the left, the values of $f(x)$ increase without bound and do not tend towards a specific value.

$\lim_{x \to 1^+} f(x) = 2$, since the graph passes through $(1, 1)$ in a continuous manner.

$\lim_{x \to -2} f(x) = 2$, since at x-values close to -2 (if not at -2 itself), the graph exhibits y-values close to 2.

$\lim_{x \to 0} f(x)$ does not exist, since the behaviors approaching $x = 0$ from the left and right sides are different.

$\lim_{x \to -\infty} f(x) = 3$, since as $x$becoems very small, we can see the graph tapering off towards the y=value of 3.

$f(0) = -1$, since the graph has a black dot (indicating a function value) at $(0, -1)$.

4. **(5 points)** *Calculate the value of* $\lim_{x \to \infty} \frac{x^4 \arctan x - 5x^3}{1 - 6x^2 - 4x^4}$*, or explicitly indicate that it does not exist.*

We may divide the numerator and denominator by the largest exponent appearing in the denominator and note that many of the terms tend towards zero:

$$\lim_{x \to \infty} \frac{x^4 \arctan x - 5x^3}{1 - 6x^2 - 4x^4} = \lim_{x \to \infty} \frac{\arctan x - \frac{5}{x}}{\frac{1}{x^4} - \frac{6}{x^2} - 4} = \lim_{x \to \infty} \frac{\arctan x - 0}{0 - 0 - 4}$$

so this limit simplifies to $\lim_{x \to \infty} \frac{\arctan x}{-4}$. The denominator is a tame constant; the numerator is a function which is known to approach $\frac{\pi}{2}$ as its parameter grows very large, so this limit has value of $\frac{\pi/2}{-4} = -\frac{\pi}{8}$.