No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(24 points)** You are building a box with a square base and an open top which will have a volume of 32000 cubic centimeters. What dimensions for this box will minimize the necessary material?

2. **(12 points)** Answer the following questions about approximation with Newton's method:

   (a) **(6 points)** Choose $x_1 = 3$ to be an initial approximation of $\sqrt[3]{30}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop x_2, a better rational approximation of $\sqrt[3]{30}$; also give an arithmetic expression (which need not **and probably should not** be simplified) for the better approximation x_3 arising from a second step of Newton's method.

   (b) **(6 points)** Starting with an initial value of 3, use two iterations of Newton's method to approximate a zero of $f(x) = x^3 - 3x^2 - 7x + 23$. Your answer need not be arithmetically simplified.

1	/ 24
2	/ 12
3	/ 23
4	/ 17
5	/ 24
6	/ (6)
Σ	/100

3. **(23 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) $\lim_{x\to 0} \frac{xe^x}{e^x-1-x}$.

   (b) $\lim_{x\to+\infty} \frac{\ln x}{\sqrt{x}}$.

   (c) $\lim_{x\to 0} \frac{e^x-e^{-x}}{\sin x}$.

   (d) $\lim_{\theta\to 0} \sin\theta \csc(5\theta)$.

   (e) $\lim_{u\to 0} \frac{u^2-\cos u}{\sqrt[3]{u}+\sin u}$.

4. **(17 points)** Answer the following questions:

   (a) **(9 points)** Find $h(x)$ given that $h'(x) = \frac{3}{x^2} + \sqrt{x}$ and $h(9) = 16$.

   (b) **(8 points)** Find the general antiderivative of $g(u) = u^3 - 5 + 6u^2 + \frac{8}{\sqrt{1-u^2}} - \frac{3}{\sqrt{u}}$.

5. **(24 points)** Answer the following questions related to the shape of the graph of the function $f(x) = e^x(x^2 - 6x + 10)$.

   (a) **(6 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.

   (b) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (c) **(4 points)** What are $f(x)$'s long term behaviors as x grows very large and as x grows very negative? Describe each direction in either words or symbols.

   (d) **(8 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

6. **(6 point bonus)** Answer the following two bonus questions on the back of this sheet.

   (a) **(3 point bonus)** Calculate $\lim_{x\to 0} \sqrt[x]{\cos x}$.

   (b) **(3 point bonus)** We have seen two procedures for estimating a difficult-to-calculate $\sqrt[n]{k}$: we can either choose a close to $\sqrt[n]{k}$ and perform a linear approximation, or choose a reasonable guess x_0 and perform Newton's method on an easy-to-manage polynomial which has $\sqrt[n]{k}$ as a zero. Prove on the back of this sheet that, in general, if we use the starting value a, a single step of Newton's method gives the exact same result as the linear approximation.