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No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(24 points)** Answer the following questions related to the shape of the graph of the function $g(x) = x^4 - 8x^2 + 8$.

(a) **(4 points)** What is $g(x)$'s long term behavior as x grows very large or very negative? Describe each direction in either words or symbols.

(b) **(6 points)** Where is $g(x)$ increasing? Where is it decreasing? Label which is which.

(c) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

(d) **(8 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

2. **(24 points)** You have 150 square inches of paper with which to design a rectangular poster. The top margin of the poster will be 2 inches, and the bottom, left, and right margins will be 1 inch. What dimensions for the poster maximize the *printable* area?

1	/ 24
2	/ 24
3	/ 13
4	/ 12
5	/ 22
Σ	/100

3. **(13 points)** Answer the following questions:

(a) **(7 points)** Find $f(x)$ given that $f'(x) = 16x^3 - 3x^2$ and $f(1) = 4$.

(b) **(8 points)** Find the general antiderivative of $h(t) = \sqrt[6]{t} + \frac{5}{t} - 2 + 4 \csc^2 t - \frac{5}{1+t^2}$.

4. **(12 points)** Answer the following questions about approximation:

(a) **(6 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^6 - 5x + 3$. Your answer need not be arithmetically simplified.

(b) **(6 points)** Choose $x_1 = 4$ to be an initial approximation of $\sqrt{17}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop x_2 , a better rational approximation of $\sqrt{17}$; also give an arithmetic expression (which need not be simplified) for the better approximation x_3 arising from a second step of Newton's method.

5. (22 points) Evaluate the following limits; if they cannot be evaluated, show why not.

(a) $\lim_{x \rightarrow 0} \frac{6x}{\arctan x}$.

(b) $\lim_{\theta \rightarrow 0} \frac{\theta + \sin \theta}{\theta + \cos \theta}$.

(c) $\lim_{u \rightarrow +\infty} \frac{e^{u/10}}{u^3}$.

(d) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2 e^x}$.

(e) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 2x + 1}$.