

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(15 points)** Determine the following limits.

(a) **(5 points)** Evaluate  $\lim_{\theta \rightarrow 0} \frac{\theta \sin \theta}{\theta^3 - \theta^2}$  or demonstrate that it cannot be evaluated.

(b) **(5 points)** Using the difference quotient, find the derivative with respect to  $x$  of  $f(x) = 6 + 3x - 2x^2$ . You may not use L'Hôpital's rule for this problem.

(c) **(5 points)** Evaluate  $\lim_{t \rightarrow \infty} \frac{t^2 - 3}{te^t + 5}$  or demonstrate that it cannot be evaluated.

2. **(20 points)** You are in charge of designing a rectangular factory, three of whose sides are bounded by walls and one of whose sides opens onto a loading dock. Safety regulations demand that all machinery be at least 3 feet from each wall and at least 2 feet from the edge of the loading dock. You need to put 3000 square feet of machinery in this factory. What dimensions for the factory will *minimize* the total ground area of the factory (i.e. both the machinery space and safety edging)?

1	/	15
2	/	20
3	/	12
4	/	15
5	/	15
6	/	15
7	/	16
8	/	18
9	/	8
10	/	16
11	/	(6)
$\Sigma$	/	150

3. **(12 points)** Consider the function  $g(x) = \frac{x^2+6x+9}{x-2}$ .
- (a) **(5 points)** Identify zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which.
- (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.
- (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.
4. **(15 points)** Let  $f(x) = e^{-2x^2}$ .
- (a) **(5 points)** Where is  $f(x)$  increasing? Where is it decreasing? Label which is which.
- (b) **(3 points)** What are the critical points of  $f(x)$ ? Is each a local maximum, a local minimum, or neither?
- (c) **(7 points)** Determine where  $f(x)$  is concave up and where it is concave down, and identify points of inflection.

5. **(15 points)** The *ophiuride* is a curve described by the equation  $x(x^2 + y^2) = 5y^2$ .
- (a) **(10 points)** Find a formula on the ophiuride for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .
- (b) **(5 points)** Find the equation of the tangent line to the ophiuride at the point  $(4, -8)$ .
6. **(15 points)** An alien spacecraft is heated to  $1800^\circ\text{F}$  by entry into the atmosphere and crash-lands on an  $100^\circ\text{F}$  day during the summer. After 20 minutes, the ship's hull has cooled to  $1400^\circ\text{F}$ .
- (a) **(5 points)** Construct a function modeling the temperature of the spacecraft  $t$  minutes after impact.
- (b) **(6 points)** The scientific survey team can begin their experiments as soon as the ship has cooled to  $400^\circ\text{F}$ . How long will they need to wait after impact to do so?
- (c) **(4 points)** How quickly is the spacecraft cooling 10 minutes after impact?

7. **(16 points)** A thirteen-foot-long ladder is leaning against a wall. The bottom of the ladder, which is currently five feet away from the wall, is slipping away from the wall at a rate of two feet per hour.

(a) **(8 points)** How quickly is the top of the ladder sliding down the wall?

(b) **(8 points)** How quickly is the angle between the ladder and the ground changing?

8. **(18 points)** Evaluate the following integrals:

(a) **(6 points)**  $\int \frac{1}{\theta} \sec^2(\ln \theta) d\theta$ .

(b) **(6 points)**  $\int_0^1 x^3 \sin(x^4 + 2) dx$

(c) **(6 points)**  $\int_1^{27} \sqrt[3]{t} dt$

9. **(8 points)** Answer the following questions about the function  $f(t) = \frac{\sqrt{2t+4}}{t-7}$ .

(a) **(4 points)** What is the domain of  $f(t)$ ?

(b) **(4 points)** Where does the derivative of  $f(t)$  exist?

10. **(16 points)** Answer the following questions:

(a) **(6 points)** Given  $h(\theta) = \frac{\sin(\theta^2)}{\ln|\theta|}$ , find  $h'(\theta)$ .

(b) **(4 points)** Find  $\int \frac{d}{dx}(\sec x - e^{\sin x})dx$ .

(c) **(6 points)** Find  $\frac{d}{dr} [\csc(r^3 - 2) \arctan(7r)]$ .