Show all work.

1. **(7 points)** *Identify the domains of the following functions:*

   (a) **(4 points)** $g(t) = \sqrt{3-t} - \ln(2+t)$.
   There are two problematic possibilities: the argument of a square-root function cannot be negative, and the argument of a logarithm cannot be non-positive. Thus, in order for this function to be evaluated, it must be the case that both $3 - t \geq 0$ and $2 + t > 0$. These can be algebraically simplified to $t \leq 3$ and $t > -2$ respectively, so the domain consists of $-2 < t \leq 3$, or, in interval notation, $(-2, 3]$.

   (b) **(3 points)** $f(x) = \frac{2x^3-5}{x^2+x-6}$.
   There is a problematic expression: the denominator of a fraction cannot be zero. Thus, in order for this function to be evaluated, it must be the case that $x^2+x-6 \neq 0$. This can be algebraically simplified, using factorization or the quadratic formula, to the requirement that $x \neq -3$ and $x \neq 2$, so the domain consists of all x not equal to -3 or 2, or, in interval notation, $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

2. **(4 points)** *This table indicates the position of a bicyclist at several points within the first 5 seconds of a race:*

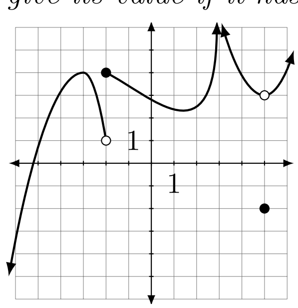
Time elapsed (in seconds)	*0.00*	*1.00*	*2.00*	*3.00*	*4.00*	*5.00*
Distance traveled (in meters)	*0.00*	*3.00*	*7.00*	*12.30*	*18.00*	*24.70*

   (a) **(2 points)** *What is the biker's average speed in the first two seconds of the race?*
   The average speed of an object with position function $f(t)$ between two times a and b is known to be $\frac{f(b)-f(a)}{b-a}$; here we are measuring the average speed between the start time (0 seconds in) and 2 seconds in, so the average speed is $\frac{f(2)-f(0)}{2-0} = \frac{7.00-0.00}{2-0} = 3.5$ meters per second.

   (b) **(2 points)** *What is the biker's average speed between the times $t = 1$ and $t = 4$?*
   We proceed as above, but here we are measuring the average speed between 1 second into the race and 4 seconds in, so the average speed is $\frac{f(4)-f(1)}{4-1} = \frac{18.00-3.00}{4-1} = 5$ meters per second.

   (c) **(3 points)** *The detailed records indicate that 4.99 seconds into the race, the bicyclist had progressed 24.635 meters. Based on this information, what would be a good estimate for the instantaneous speed after 5 seconds?*
   The exact speed is not known, but a very good estimate for the speed at the instant $t = 5$ would be the average speed between times $t = 4.99$ and $t = 5$, since that is an interval lasting a mere hundredth of a second — which is close enough to an "instant" to be pretty accurate! So we calculate $\frac{f(5)-f(4.99)}{5-4.99} = \frac{0.065}{0.01} = 6.5$ meters per second.

3. **(6 pts)** *Below is the graph of a function $f(x)$. For each of the six quantities listed to the right, give its value if it has a value, or specifically state that it does not exist.*



$f(-2)$ is 4, as evidenced by the solid dot on the graph at $(-2, 4)$.

$\lim_{x\to -2^+} f(x)$ is 4, since the curve slightly to the right of the x-value -2 is very close to height 4.

$\lim_{x\to -2^-} f(x)$ is 1, since the curve slightly to the left of the x-value -2 is very close to height 1.

$\lim_{x\to -3} f(x)$ is 4, since at x-values close to -3 (and, in fact, at -3 itself, although this is emphatically not relevant to the question) the curve is very close to height 4.

$\lim_{x\to 5} f(x)$ is 3, since at x-values close to 5 (although not at $x = 5$ itself) the curve is very close to height 3.

$\lim_{x\to 3} f(x)$ does not exist, since at x-values close to 3 the curve shoots upwards instead of tending towards a specific value. The idiomatic expression $\lim_{x\to 3} f(x) = +\infty$ is often used to describe this behavior, but in keeping with the question asked, it should be specifically stated that this limit does not exist.

4. **(2 point bonus)** *If for every value of x it is the case that $f(-x) = -f(x)$ and $g(-x) = g(x)$, what (if anything) can be said about $f(f(x))$, $f(g(x))$, $g(f(x))$, and $g(g(x))$? Justify your claims on the back of this paper.*

These conditions are what is sometimes called "function parity": the description of $f(x)$ is what is known as an "odd" function, and the description of $g(x)$ is an "even" function. The functions $f(f(x))$, $f(g(x))$, $g(f(x))$, and $g(g(x))$ also possess parity, as can be seen below:

$$f(f(-x)) = f(-f(x)) = -f(f(x))$$

$$f(g(-x)) = f(g(x))$$

$$g(f(-x)) = g(-f(x)) = g(f(x))$$

$$g(g(-x)) = g(g(x))$$

so $f(f(x))$ is odd, but the other three are even.