1. **(6 points)** *Prove that* $\lim_{x\to 2} 5-3x=-1$ *by finding a satisfactory relationship between epsilon and delta.*

   The statement $\lim_{x\to 2} 5-3x=-1$ is an assertion that, for every value $\epsilon>0$, a value δ can be furnished such that, if $0<|x-2|<\delta$, then $|5-3x-(-1)|<\epsilon$. We may justify this assertion by explicity determining how δ is calculated from ϵ to make this inference true.

$$\begin{aligned}
|5-3x-(-1)| &< \epsilon \\
|6-3x| &< \epsilon \\
|-3(x-2)| &< \epsilon \\
|-3|\cdot|x-2| &< \epsilon \\
3|x-2| &< \epsilon \\
|x-2| &< \frac{\epsilon}{3}
\end{aligned}$$

   so we may declare that the choice of δ equal to $\frac{\epsilon}{3}$ is sufficient to meet whatever challenge we are given.

2. **(4 points)** *Find a value for the parameter a such that the function* $f(x)=\begin{cases}5-x & \text{if } x\le 2\\ ax^2 & \text{if } x>2\end{cases}$ *is continuous everywhere.*

   The expressions $5-x$ and ax^2 are themselves both continuous on their domain, so the only potential problem is that the piecewise function may not be continuous at the point where it transitions between these two behaviors; thus we need simply to ensure continuity at the point $x=2$:

$$\begin{aligned}
\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) &= f(2) \\
5-2 = a\cdot 2^2 &= 5-2 \\
3 &= 4a \\
\frac{3}{4} &= a
\end{aligned}$$

3. **(4 points)** *Calculate the value of* $\lim_{x\to -\infty} \frac{2-x^3}{7x^4-9x^2}$, *or explicitly indicate that it does not exist.*

   We can look at the dominant term in the numerator and denominator, and conclude that for x of very large magnitude, $\frac{2-x^3}{7x^4-9x^2}\approx\frac{-x^3}{7x^4}$, so that

$$\lim_{x\to-\infty}\frac{2-x^3}{7x^4-9x^2}=\lim_{x\to+\infty}\frac{-x^3}{7x^4}=\lim_{x\to+\infty}\frac{-1}{7x}=0$$

   or, alternatively, we could divide the numerator and denominator by x^4 to get:

$$\lim_{x\to+\infty}\frac{2-x^3}{7x^4-9x^2}=\lim_{x\to+\infty}\frac{\frac{2-x^3}{x^4}}{\frac{7x^4-9x^2}{x^4}}=\lim_{x\to+\infty}\frac{\frac{2}{x^4}-\frac{1}{x}}{7-\frac{9}{x^2}}=\frac{0-0}{7-0}=0.$$

4. **(6 points)** *Let $f(x) = 2x^2 - 3x$. Using the difference quotient, calculate the derivative $f'(2)$.*

$$\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{[2(2+h)^2 - 3(2+h)] - (2\cdot 2^2 - 3\cdot 2)}{h} \\
&= \lim_{h\to 0} \frac{(2\cdot 2^2 + 8h + h^2 - 3\cdot 2 - 3h) - (2\cdot 2^2 - 3\cdot 2)}{h} \\
&= \lim_{h\to 0} \frac{8h + h^2 - 3h}{h} = \lim_{h\to 0} 8 + h - 3 = 5
\end{aligned}$$

5. **(2 point bonus)** *Use epsilon-delta methods (or an appropriate analogue thereof) to prove, on the back of this sheet, that $\lim\limits_{x\to 3^+} \frac{x^2-2x}{2x-6} = +\infty$.*

The statement $\lim_{x\to 3^+} \frac{x^2-2x}{2x-6} = +\infty$ is an assertion that, for every value E, a value δ can be furnished such that, if $3 < x < 3+\delta$, then $\frac{x^2-2x}{2x-6} > E$. We may justify this assertion by explicity determining how δ is calculated from E to make this inference true.

$$\begin{aligned}
\frac{x^2 - 2x}{2x - 6} &> E \\
x^2 - 2x &> (2x - 6)E \text{ if } x > 3 \\
x^2 - (2 + 2E)x + 6E &> 0 \text{ if } x > 3 \\
x &< \frac{2 + 2E - \sqrt{(2+2E)^2 - 24E}}{2} = 1 + E - \sqrt{(1+E)^2 - 6E} \text{ suffices}
\end{aligned}$$

So we note that $\delta = 1 + E - \sqrt{(1+E)^2 - 6E} - 3$ is sufficient; it's a bit fiddly to show that this quantity is in fact positive, though:

$$\begin{aligned}
(E-2)^2 &= E^2 - 4E + 4 \\
(E-2)^2 &> E^2 - 4E + 1 \\
(1 + E - 3)^2 &> E^2 + 2E + 1 - 6E \\
(1 + E - 3)^2 &> (1+E)^2 - 6E \\
1 + E - 3 &> \sqrt{(1+E)^2 - 6E} \\
\delta = 1 + E - \sqrt{(1+E)^2 - 6E} - 3 &> 0
\end{aligned}$$