Show work for all answers.

1. **(3 points)** *If* $y = 3\sqrt{t} - \frac{4}{t^3} + 5t$, *calculate* $\frac{dy}{dt}$.

   We rephrase $\sqrt{t}$ as $t^{1/2}$ and $\frac{4}{t^3}$ as $4t^{-3}$, and then invoke the power rule:

$$\frac{dy}{dt} = \frac{d}{dt}\left(3t^{1/2} - 4t^{-3} + 5t\right) = \frac{3}{2}t^{-1/2} + 12t^{-4} + 5$$

2. **(4 points)** *Calculate the second derivative* $\frac{d^2}{dx^2}\left(e^x + 2\sin x - x^4\right)$.

   First we calculate the derivative $\frac{d}{dx}\left(e^x + 2\sin x - x^4\right) = e^x + 2\cos x - 4x^3$; then we find the second derivative:

$$\begin{aligned}\frac{d^2}{dx^2}\left(e^x + 2\sin x - x^4\right) &= \frac{d}{dx}\left(e^x + 2\cos x - 4x^3\right)\\ &= e^x - 2\sin x - 12x^2\end{aligned}$$

3. **(4 points)** *If* $f(q) = \frac{q^3+e^q}{q^2-3q}$, *calculate* $f'(q)$.

   This is an application of the quotient rule:

$$\begin{aligned}f'(q) &= \frac{d}{dq}\frac{q^3+e^q}{q^2-3q}\\ &= \frac{(q^2-3q)\frac{d}{dq}(q^3+e^q) + (q^3+e^q)\frac{d}{dq}(q^2-3q)}{(q^2-3q)^2}\\ &= \frac{(q^2-3q)(3q^2+e^q) + (q^3+e^q)(2q-3)}{(q^2-3q)^2}\end{aligned}$$

4. **(5 points)** *Calculate* $\frac{d}{dx}\frac{x\sin x}{x^2-1}$.

   This is an application of the quotient rule together with the product rule:

$$\begin{aligned}\frac{d}{dx}\frac{x\sin x}{x^2-1} &= \frac{(x^2-1)\frac{d}{dx}(x\sin x) - x\sin x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}\\ &= \frac{(x^2-1)\left[\frac{d}{dx}(x)\sin x + x\frac{d}{dx}\sin x\right] - x\sin x(2x)}{(x^2-1)^2}\\ &= \frac{(x^2-1)\left[1\sin x + x\cos x\right] - x\sin x(2x)}{(x^2-1)^2}\end{aligned}$$

5. **(4 points)** *Find the equation of the tangent line to the curve* $y = 2x^3 - 3x$ *at the point* $(2, 10)$.

   We know that $\frac{dy}{dx} = 6x^2 - 3$, and in particular when $x = 2$, $\frac{dy}{dx} = 6 \cdot 2^2 - 3 = 21$. Thus, the tangent line to this curve at $(2, 10)$ has slope of 21; since it passes through $(2, 10)$, its equation can then be easily seen to be $(y - 10) = 21(x - 2)$ in point-slope form, or $y = 21x - 32$ in slope-intercept form.

6. **(2 point bonus)** *Find a general formula for* $\frac{d^3}{dx^3}[f(x)g(x)]$.

   Using the product rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

and taking the derivative of this, we see that we use the product rule twice more:

$$\begin{aligned}\frac{d^2}{dx^2}[f(x)g(x)] &= \frac{d}{dx}[f'(x)g(x)] + \frac{d}{dx}[f(x)g'(x)] \\ &= f''(x)g(x) + f'(x)g'(x) + f'(x)g'(x) + f(x)g''(x) \\ &= f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x)\end{aligned}$$

and now, to take the third derivative, we use the product rule 3 more times:

$$\begin{aligned}\frac{d^3}{dx^3}[f(x)g(x)] &= \frac{d}{dx}[f''(x)g(x)] + 2\frac{d}{dx}[f'(x)g'(x)] + \frac{d}{dx}[f(x)g''(x)] \\ &= f'''(x)g(x) + f''(x)g'(x) + 2f''(x)g'(x) + 2f'(x)g''(x) + f'(x)g''(x) + f(x)g'''(x) \\ &= f'''(x)g(x) + 3f''(x)g'(x) + 3f'(x)g''(x) + f(x)g'''(x)\end{aligned}$$