1. **(6 points)** *Given that $y \ln x = 3x \sin y$, find $\frac{dy}{dx}$ by implicit differentiation.*

We perform implicit differentiation and resolve all the derivatives, using the product rule first, and then the chain rule as necessary:

$$\begin{aligned} \frac{d}{dx}(y \ln x) &= \frac{d}{dx}(3x \sin y) \\ \left(\frac{d}{dx} y\right) \ln x + y \frac{d}{dx} \ln x &= \left(\frac{d}{dx} 3x\right) \sin y + 3x \frac{d}{dx} \sin y \\ \frac{dy}{dx} \ln x + y \cdot \frac{1}{x} &= 3 \sin y + 3x \frac{dy}{dx} \frac{d}{dy} \sin y \\ \frac{dy}{dx} \ln x + \frac{y}{x} &= 3 \sin y + 3x \frac{dy}{dx} \cos y \end{aligned}$$

And now we use algebraic techniques to isolate the term $\frac{dy}{dx}$:

$$\begin{aligned} \ln x \frac{dy}{dx} + \frac{y}{x} &= 3 \sin y + 3x \cos y \frac{dy}{dx} \\ \ln x \frac{dy}{dx} - 3x \cos y \frac{dy}{dx} &= 3 \sin y - \frac{y}{x} \\ (\ln x - 3x \cos y) \frac{dy}{dx} &= 3 \sin y - \frac{y}{x} \\ \frac{dy}{dx} &= \frac{3 \sin y - \frac{y}{x}}{\ln x - 3x \cos y} \end{aligned}$$

2. **(5 points)** *Calculate the second derivative $\frac{d^2}{dx^2} \arctan x$.*

The first derivative is a known formula: $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$; to take the second derivative we would use the quotient rule (or the chain rule):

$$\frac{d^2}{dx^2} \arctan x = \frac{d}{dx} \frac{1}{1+x^2} = \frac{(1+x^2) \cdot 0 - 1 \cdot 2x}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$$

3. **(5 points)** *Calculate the derivative $\frac{d}{dt} \arcsin(e^{4t})$.*

This is a chain rule problem: we might let $u = e^{4t}$, so that $\arcsin(e^{4t}) = \arcsin u$. Then:

$$\begin{aligned} \frac{d}{dt} \arcsin(e^{4t}) &= \frac{d}{dt} \arcsin u \\ &= \frac{du}{dt} \frac{d}{du} \arcsin u \\ &= \left(\frac{d}{dt} e^{4t}\right) \left(\frac{1}{\sqrt{1-u^2}}\right) \\ &= 4e^{4t} \cdot \frac{1}{\sqrt{1-(e^{4t})^2}} = \frac{4e^{4t}}{\sqrt{1-e^{8t}}} \end{aligned}$$

The end of the last line is an algebraic simplification and is not necessary for a correct solution.

4. **(4 points)** *Rex takes a 60mg dose of Miraclo at noon; as is ordinary for drug metabolism, the drug is depleted at a rate proportional to the amount currently in his body. At 3PM he only has 10mg of Miraclo left in his body. Construct a function $f(t)$ modeling the quantity of Miraclo in Rex's system t hours after noon.*

The above-described model for drug metabolism assumes an exponential decay function, so we know that our funtion should be $f(t) = Ce^{kt}$ for constants C and k (we know k will be negative, because this is a decay function, but that information is not of immediate use to us). Furthermore, the described information about the drug quantities at various times tells us that $f(0) = 60$ (as the initial dose is 60mg) and $f(3) = 10$ (since three hours later the dose has decayed to 10mg). Thus we may easily determine C:

$$f(0) = 60 \quad \Rightarrow \quad Ce^0 = 60 \quad \Rightarrow \quad C = 60$$

and with a little more work, may determine k:

$$\begin{aligned} f(3) &= 10 \\ 60e^{k\cdot 3} &= 10 \\ e^{3k} &= \frac{10}{60} = \frac{1}{6} \\ 3k &= \ln\frac{1}{6} \\ k &= \frac{\ln\frac{1}{6}}{3} \end{aligned}$$

so our function should be $f(t) = 60e^{\frac{\ln(1/6)}{3}t}$.