1. **(5 points)** *We are deflating a spherical balloon of radius of 10 centimeters by letting 150 cubic centimeters of air out of the balloon per second. How quickly is its radius shrinking?*

   Let us refer to the volume of this balloon as V and its radius as r. Then the above assertion that the balloon loses 150 cubic centimeters of air per second can be succinctly stated as $\frac{dV}{dt} = -150$, and the question as to how quickly the radius is changing can be related to the value of the derivative $\frac{dr}{dt}$. We also know that the radius and volume are related by the volume-of-a-sphere formula $V = \frac{4}{3}\pi r^3$. Using implicit differentiation techniques:

$$\begin{aligned}\frac{d}{dt}V &= \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)\\ \frac{dV}{dt} &= \frac{dr}{dt}\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\\ \frac{dV}{dt} &= \frac{dr}{dt}\cdot 4\pi r^2\\ \frac{\frac{dV}{dt}}{4\pi r^2} &= \frac{dr}{dt}\end{aligned}$$

   and since we know $\frac{dV}{dt} = -150$ and $r = 10$ in this scenario, we can calculate $\frac{dr}{dt} = \frac{-150}{400\pi} = \frac{-3}{8\pi}$. This result is negative because the radius is in fact decreasing as the balloon deflates.

2. **(7 points)** *Let* $f(x) = x^3 - 3x^2 + 4$. *Find the following:*

   (a) **(3 points)** *its critical points.*

   To find the critical points we inquire as to where $f'(x) = 3x^2 - 6x$ is either nonexistent or zero; the former situation will not arise since polynomials are defined everywhere, and the latter can be investigated with either the quadratic theorem or factorization:

$$\begin{aligned}3x^2 - 6x &= 0\\ 3x(x-2) &= 0\end{aligned}$$

   so $x = 0$ and $x = 2$ are critical points.

   (b) **(4 points)** *its maximum and minimum on the interval* $[-3, 1]$.

   We have to test the value of $f(x)$ at three points: the endpoints $x = -3$ and $x = 1$, and the critical point $x = 0$. We do not test the critical point $x = 2$ since it is not in the interval under consideration.

   Since $f(-3) = -27 - 27 + 4 = -50$, $f(0) = 0 - 0 + 4 = 4$, and $f(1) = 1 - 3 + 4 = 2$, we see that the minimum on the interval occurs at $x = -3$, and the maximum at $x = 4$.

3. **(4 points)** *Estimate* $(2.005)^5$ *using a well-chosen linear approximation.*

   We consider the function $f(x) = x^5$, whose derivative is $5x^4$. For x close to 2 (as 2.005 is), we can use the linear approximation:

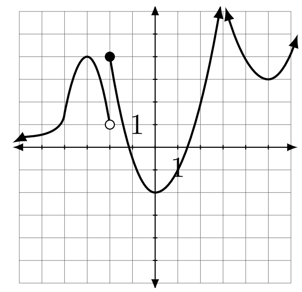
$$f(x) \approx f(2) + (x-2)f'(2)$$

   Since $f(2) = 32$ and $f'(2) = 5 \cdot 16 = 80$, it follows that

$$f(2.005) \approx 32 + 0.005 \cdot 80 = 32.4$$

   For purposes of comparison, the actual value of 2.004^5 is around 32.321283.

4. **(4 points)** *Identify, either by marking them or by giving the x-coordinates, which points on the this graph are local minima and maxima;* indicate which is which. *Also determine which, if any, points on the graph are absolute extrema on* $(-\infty, +\infty)$*; if none are, then say so.*



We may see that $x = -3$ is a local maximum, as it is higher than everything its immediate vicinity (e.g. both $x = -3.1$ and $x = -2.9$ correspond to slightly lower y-values).

The point $x = -2$ is also a local maximum. That it is slightly higher than points immediately to its right is obvious. However, it is in fact also considerably higher than points to its left (e.g. $x = -2.01$ corresponds to a y-coordinate of about 1).

The point $x = 0$ is pretty easily observed to be a local minimum, as it is lower than anything else in its immediate vicinity.

For the same reason, $x = 5$ is a local minimum.

Note that $x = 3$ is *not* a maximum: even though the approach to the point $x = 3$ gets aribrarily high, the point $x = 3$ is not in the domain of the function and thus cannot itself be an extremum.

Thus, we have 4 local extrema: maxima at $x = -3$ and $x = -2$, and minima at $x = 0$ and $x = 5$. Neither of the maxima are global, since points close to $x = 3$ are actually much higher than either of our local extrema. However, our local minimum at $x = 0$ is an absolute minimum, since no point on the graph is lower.