

1. **(7 points)** Let us consider the graph of the function $f(x) = x^4 - 2x^3$. Answer the following questions preparatory to sketching the curve.
- (a) **(2 points)** On what intervals of x -values is the function increasing? On which intervals is it decreasing? Label which is which.
- (b) **(2 points)** At what x -values are the local extrema, and which type of local extremum is each?
- (c) **(3 points)** On what intervals of x -values is the function concave up? On which is it concave down? Label which is which. Also identify the points of inflection.
2. **(7 points)** We want to enclose a rectangular animal pasture with a fence all around the outside as well as three fences parallel to a pair of sides of the pasture, so as to divide the enclosure into four rectangular sections. We have 1000 feet of fencing to use. What dimensions for our pasture maximize its area?
3. **(6 points)** Determine the value of the following limits.
- (a) $\lim_{x \rightarrow +\infty} \frac{(\ln x)^2}{x}$
- (b) $\lim_{\theta \rightarrow 0} \frac{\sin \theta - \theta}{\theta^3}$
- (c) $\lim_{t \rightarrow 0} \frac{e^t}{t \sin t + \cos t}$
4. **(2 point bonus)** You wish to make a cylindrical tin can (consisting of a circular top, a circular bottom, and a rectangle rolled to form the sides) with a volume of 350 cubic centimeters. What choice of radius and height minimize the amount of sheet metal you have to use?