

Week 1

- 1.1.13** You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.
- 1.1.17.** Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.
- 1.1.21.** A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.
- 1.1.25.** If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a + 1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a + h)$.
- 1.1.27.** Evaluate the difference quotient $\frac{f(3+h)-f(3)}{h}$ for the function $f(x) = 4 + 3x - x^2$. Simplify your answer.
- 1.1.29.** Evaluate the difference quotient $\frac{f(x)-f(a)}{x-a}$ for the function $f(x) = \frac{1}{x}$. Simplify your answer.
- 1.1.31.** Find the domain of $f(x) = \frac{x+4}{x^2-9}$.
- 1.1.33.** Find the domain of $f(t) = \sqrt[3]{2t-1}$.
- 1.1.35.** Find the domain of $h(x) = \frac{1}{\sqrt[4]{x^2-5x}}$.
- 1.1.37.** Find the domain of $F(p) = \sqrt{2 - \sqrt{p}}$.
- 1.1.47.** Find the domain and sketch the graph of $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geq 0 \end{cases}$.
- 1.1.49.** Find the domain and sketch the graph of $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$.
- 1.1.51.** Find an expression for the function whose graph is the line segment joining the points $(1, -3)$ and $(5, 7)$.
- 1.1.57.** A rectangle has perimeter 20. Express the area of the rectangle as a function of the length of one of its sides.
- 1.1.61.** An open rectangular box with volume of 2 cubic meters has a square base. Express the surface area of the box as a function of the length of a side of the base.
- 1.1.65.** In a certain state the maximum speed permitted on freeways is 65 mph and the minimum speed is 40 mph. The fine for violating these limits is \$15 for every mile per hour above or below the limits. Express the amount of the fine as a function $F(x)$ of the driving speed.
- 1.3.31.** For $f(x) = x^2 - 1$ and $g(x) = 2x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$, and identify their domains.
- 1.3.35.** For $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$, find $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$, and identify their domains.
- 1.3.41.** Express $F(x) = (2x + x^2)^4$ in the form $f \circ g$.
- 1.3.43.** Express $F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$ in the form $f \circ g$.
- 1.3.45.** Express $v(t) = \sec(t^2) \tan(t^2)$ in the form $f \circ g$.

1.5.1. Calculate $\frac{4^{-3}}{2^{-8}}$, and simplify $\frac{1}{\sqrt[3]{x^4}}$.

1.5.3. Calculate $8^{4/3}$, and simplify $x(3x^2)^3$.

1.5.19. Find the domain of the function $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$.

1.5.21. Find an exponential function whose graph passes through $(1, 6)$ and $(3, 24)$.

1.5.23. If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

1.5.29. Under ideal conditions a certain bacteria population doubles in size every 3 hours. Suppose there are initially 100 bacteria. Find the size of the population after 15 hours, and give a formula for the population after t hours.

1.6.9. Determine whether the function $f(x) = x^2 - 2x$ has an inverse.

1.6.11. Determine whether the function $g(x) = \frac{1}{x}$ has an inverse.

1.6.13. Determine whether the function $f(t)$ describing the height of a football t seconds after kickoff has an inverse.

1.6.15. Assume f is a one-to-one function. If $f(6) = 17$, then what is $f^{-1}(17)$? If $f^{-1}(3) = 2$, what is $f(2)$?

1.6.17. If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

1.6.19. The formula $C = \frac{5}{9}(F - 32)$ expresses Celsius temperature as a function of Fahrenheit temperature. Find a formula for the inverse function and interpret it.

1.6.21. Find a formula for the inverse of $f(x) = 1 + \sqrt{2 + 3x}$.

1.6.23. Find a formula for the inverse of $f(x) = e^{2x-1}$.

1.6.25. Find a formula for the inverse of $y = \ln(x + 3)$.

1.6.35. Find the exact values of $\log_5 125$ and $\log_3 \frac{1}{27}$.

1.6.37. Find the exact values of $\log_2 6 - \log_2 15 + \log_2 20$ and $\log_3 100 - \log_3 18 - \log_3 50$.

1.6.39. Write $\ln 5 + 5 \ln 3$ as a single logarithm.

1.6.41. Write $\frac{1}{3} \ln(x + 2)^3 + \frac{1}{2} [\ln x - \ln(x^2 + 3x + 2)^2]$ as a single logarithm.

1.6.51. Solve $e^{7-4x} = 6$ for x .

1.6.53. Solve $2^{x-5} = 3$ for x .

1.6.57. Find the domain of $f(x) = \ln(e^x - 3)$, and find its inverse.

1.6.61. Find the inverse of the function developed in exercise 1.5.29., and explain its meaning. Then, determine when the bacteria population reaches 50,000.

1.6.63. Find the exact values of $\sin^{-1} \frac{\sqrt{3}}{2}$ and $\cos^{-1}(-1)$.

1.6.65. Find the exact values of $\arctan 1$ and $\sin^{-1} \frac{1}{\sqrt{2}}$.

1.6.67. Find the exact values of $\tan(\arctan 10)$ and $\sin^{-1}(\sin(\frac{7\pi}{3}))$.

1.6.69. Prove that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

1.6.71. Simplify the expression $\sin(\tan^{-1} x)$.