Learning to Count

In this section you should use a *methodical* complete list of examples to determine the answer the question; even if you know other, non-listing techniques to answer the question, please explicitly list the objects being counted (if you like, you may check your answer with other techniques).

1. **(5 points)** Determine how many 4-letter strings *[edit: this said "5-letter" on the original version of the assignment. The 5-letter problem is extremely long, so it's changed]* there are using the letters "A", "B", and "C" such that each letter appears at least once in the string (e.g. "BAAC" is one such string).

2. **(5 points)** Determine how many ways there are to make change for $1.00 (100 cents) using only quarters (each worth 25 cents), nickels (each worth 5 cents), and pennies (each worth 1 cent) (e.g., "2 quarters, 6 nickels, and 20 pennies" is one such way). You may find it simpler if you use ellipsis to avoid having to write out the middle of long patterns.

3. **(5 points)** You are making up little gift bags containing 5 candies of 3 different types: chocolates, caramels and toffees, and you can use as many as you like of each type of candy. Inside the bag, of course, the candies have no intrinsic order. How many different types of gift bags are possible (e.g., "2 chocolates, 1 caramel, and 2 toffees", is one type, and "no chocolates, 4 caramels, and 1 toffee" would be another)?

4. **(5 points)** How many ways are there to express 8 as an unordered sum of odd numbers? (e.g., "7+1" is one way, and is the same as "1+7"; "1+1+1+1+1+1+1+1" is a different way).

Counting techniques

In this section, a methodical complete list will *not* in general be a practical way to answer the question. Show all work.

5. **(10 points)** Determine how many three-digit numbers (integers between 100 and 999, inclusive) there are in which all three digits are of the same parity (either all odd or all even). For instance, 402 is such a number, but 363 is not.

6. **(10 points)** A modified deck of cards contains five suits and ten cards in each suit . A "flush" is defined as in poker as a hand in which all cards are from the same suit (ignore, for the purposes of this problem, the poker convention that a "straight flush" or "royal flush" would be a different type of hand). How many different five-card hands (which are not ordered) are flushes? What is the probability that a random hand is a flush? You do not need to explicitly compute your answer, but provide an arithmetic expression which could easily be evaluated.

7. **(10 points)** Find a formula for the number of ways to build an ordered string of four letters drawn from an alphabet consisting of n letters, such that the string does not contain the same letter exactly three times.

Musica est exercitium arithmeticae occultum nescientis se numerare animi.
—Gottfried Leibniz