

Finding and using appropriate statistics

You can use either a standard combinatorial statistic (exponents, binomials, multinomials, etc.), or a combination of multiple combinatorial statistics to solve these problems; explain why you make the choices you do in solving the problem.

1. **(6 points)** How many *solutions in positive integers* are there to the equation $x+y+z = 20$, if z must be less than 10?
2. **(6 points)** I have 4 unique coins (a Double Eagle, a Seated Liberty Dollar, a Half Dime, and a Twenty Cent Piece), as well as 10 identical coins (Indian Head cents). I want to give these away to my 3 numismatic friends. How many ways are there to do so, if I may choose to give each friend as many or as few coins as I wish?

Combinatorial Proof

In this section, you will be asked to prove that two quantities are equal, by showing that they count the same family of combinatorial objects.

3. **(8 points)** Find something counted by the formula $\binom{n}{k}\binom{k}{\ell}$. Explain why you could count the same thing in a different way by using the formula $\binom{n}{\ell}\binom{n-\ell}{k-\ell}$.
4. **(10 points)** Find something counted by the formula $\sum_{i=0}^k \binom{n}{i}\binom{m}{k-i}$. Explain why you could count the same thing in a different way by using the formula $\binom{n+m}{k}$.

Pigeonholes

In this section, you will use the pigeonhole principle; when invoking it, make it clear what pigeons and pigeonholes you are using.

5. **(10 points)** Explain why it must be the case that if we choose 271 positive integers, four of them must have the same first digit and the same last digit.

Inclusion-Exclusion

In this section, you will use the inclusion-exclusion principle. Show your work.

6. **(10 points)** How many ways are there to arrange the numbers from 1 to 6 so that *at least one* number appears immediately before the next largest number? (e.g., 156342 would be such an arrangement, since it has a “56” and a “34”)

And NUH is the letter I use to spell Nutches

Who live in small caves, known as Nitches, for hutches

These Nutches have troubles, the biggest of which is

The fact there are many more Nutches than Nitches.

—Dr Seuss, *On Beyond Zebra*