Building Recurrences

1. **(6 points)** Find (but do not solve) a homogeneous linear recurrence relation, including initial conditions, for the number a_n of ways to tile a $2 \times n$ rectangle with any number of red, blue, and green dominoes.

2. **(7 points)** Find (but do not solve) a nonhomogeneous linear recurrence relation, including initial conditions, for the number b_n of ternary strings (strings of the numbers 0, 1, and 2) which *contain* the subsequence "01".

Solving recurrences

3. **(7 points)** For the sequence a_n from question 1, find a rational-function representation of the ordinary generating function $\sum_{n=0}^{\infty} a_n z^n$.

4. **(10 points)** Find a solution to the recurrence relation $a_n = 7a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 9$, $a_1 = 10$, and $a_2 = 32$, using whichever method you wish.

5. **(10 points)** Find a solution to the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ with the initial conditions $a_0 = 4$ and $a_1 = 1$.

6. **(10 points)** Recall from class the idea of a "growing annuity" where in the ith year you withdraw i dollars: we saw there that this concept with, say, an interest rate of 4% is modeled by the recurrence $a_n = 1.04a_{n-1} - n$. Solve this recurrence in terms of the initial investment a_0. How large would a_0 have to be in order for this annuity to be permanently self-perpetuating, i.e. never run out of money?

Random Fun

7. **(5 point bonus)** For F_n the Fibonacci numbers (with $F_0 = 1$ and $F_1 = 1$), show that F_n and $\sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i}$.

So, naturalists observe, a flea
Has smaller fleas that on him prey,
And these have smaller still to bite 'em,
And so proceed *ad infinitum.*

—Jonathan Swift, "On Poetry: A Rhapsody"

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