For full credit show all of your work (legibly!), unless otherwise specified. Answers need not (and probably should not) be completely reduced unless otherwise stated, and may be left in terms of sums, differences, products, quotients, exponentials, factorials, and binomial coefficients.

1. **(15 points)** Find the closed form of the recurrence relation given by initial conditions $b_0 = 4$, $b_1 = 9$, and $b_n = 6b_{n-1} - 9b_{n-2}$ for $n \geq 2$.

2. **(10 points)** Consider the following algorithm performed on a pair of numbers a and b.

   Algorithm MYSTERY(a, b):

   (1) If $a = 0$, output b.

   (2) If $b = 0$, output a.

   (3) If $a \leq b$, output the result of performing MYSTERY$(a, b - a)$.

   (4) If $a > b$, output the result of performing MYSTERY$(a - b, b)$.

   Walk through the algorithm's procedure when performed on the inputs $(60, 84)$, determining its eventual output.What does this algorithm seem to do?

1	/ 15
2	/ 10
3	/ 20
4	/ 20
5	/ 35
Σ	/100

3. **(20 points)** Find the particular solution to the recurrence relation $a_n = 4a_{n-1} + 27n$ with initial condition $a_0 = 7$.

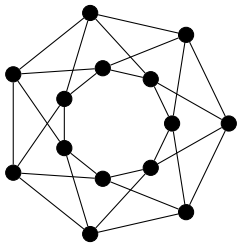
4. **(20 points)** The following question relates to algorithm design.

   (a) **(15 points)** Write an algorithm, using only simple computational steps, which takes as input a list of numbers $a_1, a_2, \ldots, a_n$, and outputs the largest number in the list (note that the elements of the list will generally not be in order, so the last element may not be the largest).

   Input: sequence $a_1, a_2, \ldots, a_n$
   Output: the maximum value in the sequence

   (b) **(5 points)** Justify and state a good asymptotic bound in big-O notation on the number of steps taken by your algorithm.

5. **(35 points+5 point bonus)** Let G be the graph shown below; label vertices as necessary.



(a) **(8 points)** Is this graph Eulerian? Explain why or why not.

(b) **(15 points)** Using both an explicit coloring and an argument, show that $\chi(G)$ is exactly 3.

(c) **(12 points)** Demonstrate that this graph has a subgraph isomorphic to C_{14}. What property follows from the existence of such a subgraph?

(d) **(5 point bonus)** Is this graph planar? Either give an explicit planar representation or explain your reasoning.