

1. (12 points)

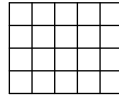
(a) (3 points) How many even four-digit numbers have at least one 7 appearing as a digit?

(b) (3 points) How many even four-digit numbers have at least one 4 appearing as a digit?

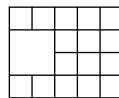
(c) (6 points) How many even four-digit numbers have a 7 or a 4 (or both) appearing as digits?

2. (12 points)

(a) (4 points) How many direct paths are there through the following two-dimensional grid?



(b) (8 points) How many direct paths are there through the following two-dimensional grid?



3. (12 points)

(a) (9 points) Prove by induction that  $2^n < (n - 1)!$  for  $n \geq 6$ .

(b) (3 points) Evaluate  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{67 \cdot 68}$

4. (12 points)

(a) (4 points) What is the coefficient of  $x^5yz^2$  in the expansion of  $(2x + 3y + 4z)^8$ ?

(b) (8 points) Assuming  $n$  is odd, evaluate

$$\binom{n+1}{1} + \binom{n+1}{3} + \binom{n+1}{5} + \binom{n+1}{7} + \cdots + \binom{n+1}{n}$$

## 5. (12 points)

- (a) (4 points) Agamemnon, Brunhilde, Cihuacoatl, and Daikoku are dividing up a pot containing 8 identical gold coins. In how many ways can they do so?
- (b) (4 points) How many anagrams (rearrangements of letters) are there for the word “AGAMEMNON”? The anagrams need not be actual English words.
- (c) (4 points) How many anagrams of “AGAMEMNON” do not have the “A”s next to each other?