For full credit show all of your work (legibly!), unless otherwise specified. Answers need not (and probably should not) be completely reduced unless otherwise stated, and may be left in terms of sums, differences, products, quotients, exponentials, factorials, and binomial coefficients.

1. **(15 points)** Computationally, a vector is simply a list of numbers. We may represent an n-dimensional vector $\vec{a}$ as a list of n coordinates $(a_1, a_2, a_3, \ldots, a_n)$.

   (a) **(10 points)** Write an algorithm, using only simple computational steps, to compute the dot product of the vectors $\vec{a}$ and $\vec{b}$. Recall that a dot product of two vectors is the sum of the coordinatewise products, e.g. $(5,3,1,-2)\cdot(-1,0,4,3) = 5\cdot -1 + 3\cdot 0 + 1\cdot 4 + (-2)\cdot 3 = -7$.

   Input: sequences $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$
   Output: number c which is equal to $\vec{a}\cdot\vec{b}$

   (b) **(5 points)** Justify and state a good asymptotic bound in big-O notation on the number of steps taken by your algorithm.

2. **(10 points)** Find the closed form of the recurrence relation given by initial conditions $a_0 = 5$, $a_1 = 0$, and $a_n = 2a_{n-1} + 24a_{n-2}$ for $n \geq 2$.

3. **(10 points)** Consider the following algorithm performed on a sequence of numbers $a_1, a_2, \ldots, a_n$.

   (1) Let $i = 1$.

   (2) Let $q = i$ and let $j = q + 1$.

   (3) If $a_j < a_q$, then let $q = j$.

   (4) Increment j.

   (5) If $j \leq n$, then return to step (3).

   (6) Swap the values of a_i and a_q (if $i = q$, do nothing).

   (7) Increment i.

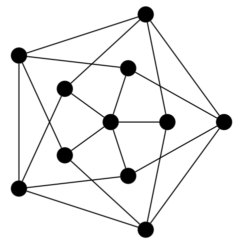
   (8) If $i < n$, then return to step (2).

   (a) **(4 points)** Walk through the algorithm's procedure when performed on the 5-term sequence $(4, 8, 1, 10, 2)$. What does this algorithm seem to do?

   (b) **(6 points)** Give a big-O estimate of the number of operations, in terms of n, which this algorithm takes to perform its task.

4. **(12 points)** Find the particular solution to the recurrence relation $b_n = 4b_{n-1} + 21b_{n-2} - 25 \cdot 2^n$ with initial conditions $b_0 = 1$ and $b_1 = 27$.

5. **(20 points+5 point bonus)** Let G be the graph shown below; label vertices as necessary.



(a) **(10 points)** Demonstrate via an explicit coloring that $\chi(G) \leq 4$, and give an argument that $\chi(G) > 2$.

(b) **(5 points)** Is this graph Eulerian? Explain why or why not.

(c) **(5 points)** Demonstrate that this graph has a subgraph isomorphic to C_6.

(d) **(5 point bonus)** Is this graph planar? Either give an explicit planar representation or explain your reasoning.