1. **(12 points)** Prove that for any integers a, b, c, and n, if $a \equiv b \pmod{n}$, then $ac \equiv bc \pmod{n}$.

2. **(17 points)** For each natural number i, let A_i be the set of integers $\{i, i+1, i+2, \ldots, (i+1)^2\}$. Calculate the results of the following indexed set operations:

   (a) $\bigcup_{i=1}^{3} A_i$.

   (b) $\bigcap_{i=1}^{3} A_i$.

   (c) $\bigcup_{i=1}^{\infty} A_i$.

   (d) $\bigcap_{i=1}^{\infty} A_i$.

3. **(20 points)** Prove that for any positive integer n, it is the case that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

4. **(18 points)** For each of the following relations, determine whether it is reflexive, symmetric, and/or transitive, providing a brief explanation for the properties which hold and a counterexample for properties which do not hold.

   (a) The relation R_1 on $\mathbb{R}$ given by $x \; R_1 \; y$ iff $|x - y| = 1$.

   (b) The relation R_2 on $\{0, 1, 2\}$ given by $R_2 = \{(0,0), (0,1), (1,1), (1,2), (2,0), (2,1), (2,2)\}$.

   (c) The relation R_3 on $\mathbb{N}$ given by $x \; R_3 \; y$ iff $\gcd(x, y) \neq 1$ (i.e., if x and y have a factor in common).

5. **(18 points)** Give examples of sets satisfying the following conditions, or explain why they cannot be met:

   (a) sets A and B such that $A \in \mathcal{P}(B)$ and $B \in \mathcal{P}(A)$.

   (b) nonempty sets R, S, and T such that $R \subseteq S$ and $S = R - T$.

   (c) nonempty sets X, Y, and Z such that $X \subseteq Y \subseteq Z$ and $(Z - X) \cap Y = \emptyset$.

6. **(15 points)** Prove that for sets A, B, and C, $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.