

For a nonempty set S , let $f : S \rightarrow S$ be a function. Note that since $f \subseteq S \times S$, f can *also* be regarded as a relation.

- Prove that if f is not the identity function $f(x) = x$, then f is non-reflexive when considered as a relation on S .
- Prove that f is symmetric when considered as a relation on S if and only if $f(f(x)) = x$ for every $x \in S$.
- **Bonus problem:** demonstrate that some functions other than the identity may be transitive when considered as a relation on S . Characterize all such functions.