1. **(17 points)** For each natural number i, let S_i be the half-open interval $[1, 2 + \frac{1}{i})$. Calculate the results of the following indexed set operations:

   (a) $\bigcap_{i=1}^{3} S_i$.

   (b) $\bigcup_{i=1}^{3} S_i$.

   (c) $\bigcap_{i=1}^{\infty} S_i$.

   (d) $\bigcup_{i=1}^{\infty} S_i$.

2. **(12 points)** Prove or disprove: for sets A and B, if $A \cap B = A$, then $A \subseteq B$.

3. **(18 points)** Give examples of sets satisfying the following conditions, or explain why they cannot be met:

   (a) sets A, B, and C such that $A \subsetneq B$, $A \in C$, and $B \in C$.

   (b) sets S and T such that $T = \mathcal{P}(S)$ and $S \cap T \neq \emptyset$.

   (c) sets X, Y, and Z such that $X \cup Y = Z$ and $Z \subsetneq X$.

4. **(15 points)** Prove that if a, b, m, and n are integers such that $m \mid n$ and $a \equiv b \pmod{n}$, then $a \equiv b \pmod{m}$.

5. **(18 points)** For each of the following relations, determine whether it is reflexive, symmetric, and/or transitive, providing a brief explanation for the properties which hold and a counterexample for properties which do not hold.

   - The relation R_1 on $\{1, 2, 3\}$ given by $R_1 = \{(1, 1), (1, 2), (2, 2)\}$.
   - The relation R_2 on $\mathbb{R}$ given by $x \ R_2 \ y$ iff $|x| \geq |y|$.
   - The relation R_3 on $\mathcal{P}(\mathbb{N})$ given by $A \ R_3 \ B$ iff $A \cap B \neq \emptyset$.

6. **(20 points)** Prove that for any positive integer n, it is the case that $1(1!) + 2(2!) + \cdots + n(n!) = (n + 1)! - 1$. Recall that $k! = k(k - 1)(k - 2) \cdots (3)(2)(1)$.