

1. Prove that for every integer  $n$ ,  $n^3 \equiv n \pmod{6}$ .
2. Let us say that, for nonzero rational numbers  $a$  and  $b$ ,  $a \simeq b$  if  $a$  and  $b$  have the same denominator when written in lowest terms. Explain why  $\simeq$  is an equivalence relation and describe its equivalence classes.
3. Prove that for every number  $k$  there is a number  $N$  such that all of the numbers  $N + 1, N + 2, N + 3, \dots, N + k$  are composite.
4. Determine the symmetries of a rectangle which is not a square; give each symmetry a name and produce a Cayley table.
5. Describe the infinite group of symmetries of the following infinitely long figure. Determine as many rules as you can for multiplying the symmetries, and determine whether this group is Abelian.

