

1. Identify each of the following algebraic structures as either a group or not a group, and justify your answer. For each structure which is a group explain if it is Abelian.
 - The real numbers \mathbb{R} under the operation $a \odot b = a + b + 1$.
 - The integers \mathbb{Z} under the operation $a \max b$, which returns the larger of a and b (e.g. $7 \max -3 = 7$).
 - The set of subsets of $\{1, 2, 3, 4, 5\}$ under the union operation $S \cup T$.
 - The set of strings of the symbols a , b , and b^{-1} under the operation of concatenation with the rule that two adjacent as or bs cancel, e.g. $(ababa)(ab) = ababaab = abab\cancel{ab} = aba$.
2. Let D_6 be the group of symmetries of a hexagon. Identify a subgroup of D_6 with each of the following orders: 1, 2, 3, 6.
3. Prove that if G is a finite group with order 7, then G is cyclic.
4. Let G be a group such that for any $a, b, c, d, x \in G$, if $axb = cxd$, then $ab = cd$. Prove that G is Abelian.
5. Which elements of Z_{200} have order 5? Explain how you know.