

1. Prove that  $(1234)$  cannot be written as the product of 3-cycles.
2. Find an element of  $S_9$  with order 20, and prove that no element of  $S_8$  has order 20.
3. Prove that  $S_4$  is not isomorphic to  $D_{12}$ .
4. Recall that the *centralizer* of an element  $a$  of a group, denoted,  $C(a)$ , is the subgroup consisting of all  $b$  such that  $ab = ba$ .  
Prove that for any  $a, g \in G$ ,  $C(a)$  is isomorphic to  $C(gag^{-1})$ .
5. Describe  $\text{Aut}(D_5)$  as fully as possible: identify its order, its elements, and any multiplicative rules you can find.