1. Let G be a finite group whose order is greater than 2 but not divisible by 4. Prove that G contains an element of odd order (other than the identity).

2. Let G be an Abelian group of order 15. Prove that G is cyclic.

3. Suppose $|G| = p^2$ with p prime. For non-identity elements a and b of G, prove that if $\langle a \rangle \neq \langle b \rangle$, then every element of the group can be expressed in the form $a^k b^\ell$ for integers k and ℓ.

4. Recall that the *center* $Z(G)$ of a group G is the subgroup whose elements are of the form $\{x \in G : xa = ax \text{ for all } a \in G\}$. Prove that $Z(G_1 \oplus G_2) = Z(G_1) \oplus Z(G_2)$; that is, that the center of a direct product is the direct product of centers.

5. Prove that $S_3 \oplus Z_2$ is isomorphic to D_6; you may find it helpful to define a specific isomorphism.

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