1. Prove that if N and M are normal subgroups of G, then NM is also a normal subgroup of G.

2. For a group G, let $S = \{x^{-1}y^{-1}xy : x, y \in G\}$ and let H be the subgroup of G generated by S. Prove that H is a *normal* subgroup of G and that G/H is Abelian.

3. Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$ with multiplication rules such that negation works as it normally does, and $i^2 = j^2 = k^2 = -1$, and $ij = k$, $ji = -k$, $jk = i$, $kj = -i$, $ki = j$, and $ik = -j$ (This group is known as the *quaternion group*; it is something of a crossbreed of complex numbers and vector cross products). Show that the quaternion group can *not* be written as an internal direct product of any nontrivial subgroups.

4. Prove (with homomorphisms or directly) that $D_{2n}/\langle r^n \rangle \simeq D_n$.

5. Let G be a group such that for certain primes p and q, there is a homomorphism from G *onto* (i.e. surjectively mapping to) Z_{pq}. Prove that G has normal subgroups of index p and q.

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