This problem set is all for bonus points: each question is worth 8 points, so the total may be worth as much as 24 points.

1. Prove that every group of order 20 has exactly four elements of order 5 (note: this is trivial to demonstrate for individual groups you know, like Z_{20} and D_{10}; the goal is to prove that it is universally true).
2. Let G be a group of order 60 with no normal subgroups other than $\{e\}$ and G itself. Determine, with explanation, how many Sylow 2-, 3-, and 5-subgroups G has.
3. Prove that, if $|G| = 56$, then G has at least one normal subgroup other than $\{e\}$ and G itself.

due Tuesday, December 9, 2014