1. **(15 points)** Either prove the following conjecture, or furnish a counterexample.

   Conjecture 1. *For any group G, subgroup H of G, and element k of G, the set $S = \{khk^{-1} : h \in H\}$ is a subgroup of G.*

2. **(15 points)** Let H_1 and H_2 be subgroups of a group G such that H_1 is not a subgroup of H_2 and H_2 is not a subgroup of H_1. Prove that the set $H_1 \cup H_2$ is not a group.

3. **(15 points)** The following questions relate to the additive group modulo 30, known variously as either Z_{30} or $(\mathbb{Z}_{30}, +)$

   (a) **(5 points)** Which, if any, elements of this group have order 12? Explain your reasoning.

   (b) **(5 points)** How many subgroups does Z_{30} have, including itself and the trivial one-element subgroup?

   (c) **(5 points)** List all the generators of this group.

4. **(20 points)** Identify each of the two following algebraic structures as either a group or not as a group. Explain your reasoning and, if the structure in question is a group identify it as Abelian or non-Abelian.

   (a) The set of subsets of $\{1, 2, 3, 4, 5\}$ under the operation $\triangle$ of symmetric difference, given by the rule $S \triangle T = (S - T) \cup (T - S)$, e.g. $\{1, 2, 4\} \triangle \{2, 4, 5\} = \{1, 5\}$.

   (b) The set of ordered pairs of integers under the operation $*$ defined by the rule $(a, b) * (c, d) = (ac + ad, bd)$.

5. **(20 points)** Answer the following questions about symmetric groups.

   (a) **(8 points)** How many odd permutations are there in S_5? Explain your reasoning.

   (b) **(6 points)** What is the order of the following permutation?

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 8 & 9 & 4 & 2 & 1 & 7 & 6 & 3 \end{bmatrix}$$

   (c) **(6 points)** Is the permutation from the previous part odd or even? Explain why.

6. **(15 points)** Find a subgroup H of the dihedral group D_9 such that $|H| = 6$ (note that $|D_9| = 18$). Demonstrate that D_9 does not contain an *Abelian* subgroup of order 6.