1. **(15 points)** Prove that, if G is a finite group of odd order, then the product of all the elements of G is the identity.

2. **(20 points)** Below, let $N \trianglelefteq G$ and $H \leq G$.

   (a) **(15 points)** Prove that $(N \cap H) \trianglelefteq H$.

   (b) **(5 points)** Demonstrate by example (specific choices of G, N, and H) that it might *not* be the case that $(N \cap H) \trianglelefteq G$.

3. **(15 points)** Answer the following questions about direct products.

   (a) **(5 points)** Is $Z_4 \oplus Z_3$ isomorphic to Z_{12}? Explain your reasoning.

   (b) **(10 points)** Groups G and H have orders 8 and 9 respectively. Prove that $G \oplus H$ contains an element of order divisible by 6. (Please note that G and H need not be cyclic or even Abelian.)

4. **(15 points)** For each group G and subgroup H listed below, identify H as either a normal subgroup or not a normal subgroup of G, and explain why:

   (a) **(5 points)** $G = D_{12} \oplus Z_2$, $H = \langle r \rangle \oplus Z_2$.

   (b) **(5 points)** $G = S_4$, $H = \{e, (12), (34), (12)(34)\}$.

   (c) **(5 points)** $G = Z$, $H = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$.

5. **(20 points)** Answer the following questions about homomorphisms.

   (a) **(5 points)** Identify the kernel and image of the homomorphism from D_4 to $Z_2 \oplus Z_4$ (the infinite cyclic group) given by the rules $\varphi(r) = (1, 0)$ and $\varphi(f) = (0, 2)$.

   (b) **(10 points)** Show that no homomorphism from D_4 to Z_4 is surjective.

   (c) **(5 points)** Using the definition of φ from part (a), what well-known group is $G/\ker\varphi$ isomorphic to?

6. **(15 points)** Prove that if $N \trianglelefteq G$ and $M \trianglelefteq G$, then $N \cap M \trianglelefteq G$.