1. **(15 points)** Answer the following questions about direct products.
   (a) **(5 points)** Is $Z_6 \oplus Z_3$ isomorphic to Z_{18}? Explain your reasoning.
   (b) **(10 points)** An element of $G \oplus H$ has order 12. Given that G and H are finite groups, prove that the order of either G or H is divisible by 4.

2. **(15 points)** Give an example of a group G and subgroups H and K such that HK is not a subgroup of G.

3. **(20 points)** Let $H \trianglelefteq G$, and let $K = \{k \in G : kh = hk \text{ for all } h \in H\}$. Prove that $K \trianglelefteq G$.

4. **(20 points)** Answer the following questions about homomorphisms.
   (a) **(5 points)** Identify the kernel and image of the homomorphism from D_6 to Z_{12} given by the rules $\varphi(r) = 2$ and $\varphi(f) = 0$.
   (b) **(10 points)** Show that there are exactly two homomorphisms from D_6 to Z_{12} which map f to an element other than zero.
   (c) **(5 points)** Using the definition of φ from part (a), what well-known group is $G/\ker\varphi$ isomorphic to?

5. **(15 points)** Prove that if G is a group with order p^k for prime p and some positive integer k, then G contains an element of order p.

6. **(15 points)** For each group G and subgroup H listed below, identify H as either a normal subgroup or not a normal subgroup of G, and explain why:
   (a) **(5 points)** $G = Z_{10} \oplus Z_{10}$, $H = \langle(2, 2)\rangle$.
   (b) **(5 points)** $G = D_6$, $H = \{e, f\}$.
   (c) **(5 points)** $G = S_5$, $H = A_5$.