

Show all work.

1. **(20 points)** Two models of guitar are being produced by a factory. Model A has a labor cost of \$30 and a material cost of \$20, while Model B has a labor cost of \$40 and a material cost of \$30. Over the course of three weeks, you have the following budget allocations available:

	Week 1	Week 2	Week 3
Labor	\$1800	\$1750	\$1720
Material	\$1200	\$1250	\$1280

Determine how many guitars of each type should be produced each week.

2. **(10 points)** Use the substitution method to solve the system of equations 
$$\begin{cases} 7m + 12n = -1 \\ 5m - 3n = 7 \end{cases}$$

3. **(10 points)** For each of the following matrices indicate whether it is in reduced form; if it is not, explain why and indicate the row operation which would put it into reduced form.

(a) 
$$\left[ \begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 0 & -1 \end{array} \right]$$
.

(b) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$
.

(c) 
$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$
.

4. **(15 points)** Either calculate the inverse of  $\begin{bmatrix} 2 & -2 & 4 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$  or demonstrate that it has no inverse.

5. **(15 points)** Perform the following calculations, or explain why they cannot be performed.

(a)  $\begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \end{bmatrix}$ .

(b)  $3 \begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix} - 2 \begin{bmatrix} -2 & 6 \\ 5 & -3 \end{bmatrix}$ .

(c)  $\begin{bmatrix} 1 & 2 & 0 \\ 5 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 5 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ .

6. **(10 points)** A jar contains pennies (worth 1 cent), nickels (worth 5 cents), and dimes (worth 10 cents). Pennies weigh 2.5 grams, nickels weigh 5 grams, and dimes weigh 2 grams. There are 281 coins in the jar, with a value of 1316 cents and a weight of 792 grams. We wonder how many of each coin are in the jar.

Convert the situation described into a system of equations which would help us answer our question. Indicate what each of your variables means. *You do not need to solve the system.*

7. **(20 points)** Use the knowledge that  $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & -12 & 3 \\ -2 & -4 & 1 \\ 2 & 5 & -1 \end{bmatrix}$  to help solve the following questions.

(a) Find a solution to the system of equations 
$$\begin{cases} x - 3y & = & 3 \\ y + z & = & -2 \\ 2x - y + 4z & = & 0 \end{cases}$$

(b) Find a matrix  $X$  such that 
$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 4 \end{bmatrix} X - \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}.$$