

Show all work.

1. **(20 points)** For each of the following propositions, construct a truth table and explicitly identify the proposition as either a tautology, a contradiction, or neither.

(a) **(5 points)** $(p \rightarrow q) \wedge (p \wedge q)$

(b) **(5 points)** $p \rightarrow (p \vee \neg q)$

(c) **(10 points)** $[(p \wedge q) \vee (\neg p \wedge r)] \rightarrow (q \vee r)$.

2. **(10 points)** If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 5, 7\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, determine the results of each of the following operations.

(a) $A \cup B$.

(b) $A \cap C$.

(c) A' .

(d) $B \cap C$.

(e) $B \cap C'$.

(f) $B \cap C'$.

(g) $(A \cup B)'$.

(h) $A \cup B \cup C$.

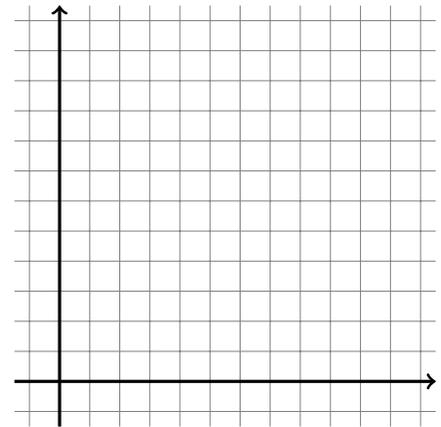
(i) $B \cup C$.

(j) $B' \cup C'$.

3. **(40 points)** A special diet for laboratory animals must contain at least 850 units of vitamins, 800 units of minerals, and 1150 calories. Two feed mixes are available: the light mix, of which each gram costs \$0.04 and contains 2 units of vitamins, 2 units of minerals, and 4 calories; and the heavy mix, of which each gram costs \$0.09 and contains 5 units of vitamins, 4 units of minerals, and 5 calories. How much of each mix should you purchase to minimize the cost?

(a) **(10 points)** Formulate a linear programming problem describing the scenario above. Identify what your variables represent, and clearly label constraints and the objective function.

(b) **(20 points)** Solve the above linear programming problem.



(c) **(10 points)** The price of the heavy mix decreases to \$0.06 per gram. How does this change the solution you determined above?

4. **(15 points)** Determine the number of ways to do each of the following things:
- (a) **(3 points)** select one course from each field of study if there are 2 courses available in history, 3 in science, 2 in math, 2 in philosophy, and 1 in English.

 - (b) **(3 points)** select one course *in total* if there are 2 courses available in history, 3 in science, 2 in math, 2 in philosophy, and 1 in English.

 - (c) **(4 points)** select a three-digit number where all three digits are different and none of the digits is zero.

 - (d) **(5 points)** select four committee members from a group of 9 people.
5. **(15 points)** Determine the probability of each of the following events:
- (a) **(8 points)** Three cards drawn from a standard 52-card deck are all spades.

 - (b) **(7 points)** Rolling two fair 6-sided dice results in a sum of 7 or 11.