1. **(7 points)** *A fruit grower uses two brands of fertilizer. Each bag of brand A contains 8 pounds of nitrogen and 4 pounds of phosphoric acid, while each bag of brand B contains 7 pounds of nitrogen and 6 pounds of phosphoric acid. Tests indicate that the grove needs 720 pounds of nitrogen and 500 pounds of phosphoric acid. Set up a system of equations to help determine how many bags of each brand they will need, and indicate what your variables represent.* You do not need to solve the system you set up!

   We may let x be the number of bags used of brand A, and y the number of bags of brand B. Then, our x bags of brand A and y bags of brand B yield $8x$ pounds and $7y$ pounds respectively of nitrogen; thus we want $8x + 7y$ to be equal to 720 to supply the desired nitrogen. Likewise, our x bags of brand A and y bags of brand B yield $4x$ pounds and $6y$ pounds respectively of phosphoric acid, so we want $4x + 6y$ to be 500. Our system is thus

$$\begin{cases} 8x + 7y = 720 \\ 4x + 6y = 500 \end{cases}.$$

2. **(6 points)** *Solve the following system of equations using* substitution*:*

$$\begin{cases} 2x + y = 6 \\ x - y = -3 \end{cases}$$

   There are four different approaches; we include all four here for completeness.

   We could solve the first equation for x, finding that $x = \frac{6-y}{2}$, and substitute it into the second to get

$$\begin{aligned} \frac{6-y}{2} - y &= -3 \\ 3 - \frac{3}{2}y &= -3 \\ -\frac{3}{2}y &= -6 \\ y &= 4 \end{aligned}$$

   so $y = 4$, and then $x = \frac{6-4}{2} = 1$.

   Alternatively, we could solve the first equation for y, finding that $y = 6 - 2x$, and substitute it into the second to get

$$\begin{aligned} x - (6 - 2x) &= -3 \\ 3x - 6 &= -3 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

   so $x = 1$, and then $y = 6 - 2 \cdot 1 = 4$.

   If we chose to solve the second equation for x, finding that $x = y - 3$, we would substitute it into the first to get

$$\begin{aligned} 2(y - 3) + y &= 6 \\ 3y - 6 &= 6 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

so $y = 4$, and then $x = 4 - 3 = 1$.

Lastly, if we solved the second equation for y, finding that $y = x + 3$, we would substitute it into the first to get

$$\begin{aligned} 2x + (x + 3) &= 6 \\ 3x + 3 &= 6 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

so $x = 1$, and then $y = 1 + 3 = 4$.

Regardless of which approach you use, you will find that $x = 1$ and $y = 4$.

3. **(7 points)** *Solve the following system of equations using* augmented matrix methods*:*

$$\begin{cases} 3x_1 - x_2 = 2 \\ x_1 + 2x_2 = 10 \end{cases}$$

Pulling coefficients and constants from the equations gives us the augmented matrix $\left[\begin{array}{cc|c} 3 & -1 & 2 \\ 1 & 2 & 10 \end{array}\right]$. We shall now perform row-operations to get a simpler matrix:

$$\begin{aligned} \left[\begin{array}{cc|c} 3 & -1 & 2 \\ 1 & 2 & 10 \end{array}\right] &\sim \left[\begin{array}{cc|c} 1 & 2 & 10 \\ 3 & -1 & 2 \end{array}\right] && (R_1 \leftrightarrow R_2) \\ &\sim \left[\begin{array}{cc|c} 1 & 2 & 10 \\ 0 & -7 & -28 \end{array}\right] && (R_2 - 3R_1 \to R_2) \\ &\sim \left[\begin{array}{cc|c} 1 & 2 & 10 \\ 0 & 1 & 4 \end{array}\right] && (R_2 \div (-7) \to R_2) \\ &\sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 4 \end{array}\right] && (R_1 - 2R_2 \to R_1) \end{aligned}$$

so $x_1 = 2$ and $x_2 = 4$.