1. **(10 points)** *Either calculate the result of each arithmetic operation, or explain why it cannot be performed.*

   (a) **(4 points)** $\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}^2$

   Squaring a matrix is taking a product, and it is possible in this case:

$$\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} (-3)(-3) + 1 \cdot 2 & (-3)1 + 1 \cdot 5 \\ 2(-3) + 5 \cdot 2 & 2 \cdot 1 + 5 \cdot 5 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ 4 & 27 \end{bmatrix}$$

   (b) **(6 points)** $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} - 2 \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$

   We can perform the matrix and scalar products, and then subtract the results:

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} - 2 \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ -2 & -8 \end{bmatrix} - \begin{bmatrix} -6 & 2 \\ 4 & 10 \end{bmatrix} = \begin{bmatrix} 15 & 1 \\ -6 & -18 \end{bmatrix}$$

2. **(3 points)** *Test to see if* $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ *and* $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ *are inverses.*

   We calculate the product to find that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   so since their product is the identity matrix, they are in fact each other's inverses.

3. **(7 points)** *Find the inverse, if it exists, of the matrix* $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 2 & -1 & 4 \end{bmatrix}$.

   We augment this matrix with an identity matrix, and perform Gauss-Jordan reduction:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & -1 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 5 & 4 & -2 & 0 & 1 \end{array}\right] \qquad (R_3 - 2R_1 \to R_3)$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & -5 & 1 \end{array}\right] \qquad \begin{pmatrix} R_1 + 3R_2 \to R_1 \\ R_3 - 5R_2 \to R_3 \end{pmatrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 & -1 \end{array}\right] \qquad (R_3 \times (-1) \to R_3)$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -12 & 3 \\ 0 & 1 & 0 & -2 & -4 & 1 \\ 0 & 0 & 1 & 2 & 5 & -1 \end{array}\right] \qquad \begin{pmatrix} R_2 - \ \ R_3 \to R_2 \\ R_3 - 3R_3 \to R_2 \end{pmatrix}$$

   so $\begin{bmatrix} -5 & -12 & 3 \\ -2 & -4 & 1 \\ 2 & 5 & -1 \end{bmatrix}$ is the desired inverse.