1. **(7 points)** *Formulate a linear programming problem describing the following question. Clearly label what each of your variables represents, and identify the constraints and an objective function.*

   You are renting buses and vans to transport high school students on a trip. Each bus can transport 40 students, requires 3 chaperones, and costs \$1200 to rent. Each van can transport 8 students, requires one chaperone, and costs \$100 to rent. You must have enough transport for all 400 students in the class, and can use no more than 36 chaperones. How many vehicles of each type should you rent to minimize transportation costs?

   We may let x be the number of buses we rent, and y the number of vans. Then, our x buses and y vans can respectively transport $40x$ and $8y$ students; thus we want $40x + 8y$ to be at least 400, in order to provide seats to all 400 students. Likewise, our x buses and y vans need $3x$ and y chaperones respectively, so we want $3x + y$ to be no more than 36, since we have a maximum of 36 chaperones. Thus, our constraints are

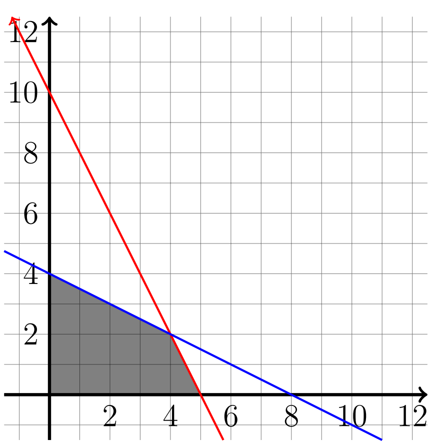
$$\begin{cases} 40x + 8y \geq 400 \\ 3x + y \leq 36 \\ x, y \geq 0 \end{cases}$$

   Finally, our goal is to minimize costs. Since each bus costs \$1200 and each van costs \$100, the total cost of renting x buses and y vans is $1200x + 100y$, which is our objective function.

   For reference, the answer to the question you were asked to restate is 7 buses and 15 vans, but that was not something we were asked to find in this question.

2. **(13 points)** *Find the minimum and maximum values of $z = 5x + 5y$ subject to the conditions*
$$\begin{cases} 2x + y \leq 10 \\ x + 2y \leq 8 \\ x, y \geq 0 \end{cases}.$$

   The feasible region is as shown below:



   Three corners of this region are easily determined to be $(0, 0)$, $(5, 0)$, and $(0, 4)$. The fourth can be found either by inspection of the graph or by solving the system of equations

$$\begin{cases} 2x + y = 10 \\ x + 2y = 8 \end{cases}$$

One way to solve this might be to substitute the first equation, rewritten as $y = 10 - 2x$, into the second equation, to get $x+2(10-2x) = 8$, or $-3x = -12$, so $x = 4$. Then $y = 10-2\cdot 4 = 2$. Thus the fourth point is $(4, 2)$.

Armed with this fact, we can now simply test the four corners to see where the objective function reaches extreme values:
At $(0, 0)$, $5x + 5y = 0$.
At $(5, 0)$, $5x + 5y = 25$.
At $(4, 2)$, $5x + 5y = 30$.
At $(0, 4)$, $5x + 5y = 20$.

Thus, the minimum value is 0, achieved at $(0, 0)$, and the maximum is 30, achieved at $(4, 2)$.