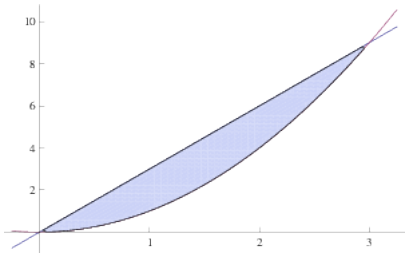
This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. While integrals must be fully calculated, it is not necessary to arithmetically or algebraically simplify the results: common trigonometric calculations, however, must be completely evaluated.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(15 points)** The region shown below is the area between the curves $y = 3x$ and $y = x^2$. Find the center of mass of this region.



1	/15
2	/15
3	/15
4	/15
5	/10
6	/15
7	/20
Σ	

2. **(15 points)** Evaluate the following integrals, or if they cannot be evaluated, demonstrate why not.

   (a) **(7 points)** $\int_{-2}^{4} \frac{1}{x} dx$

   (b) **(8 points)** $\int_{5}^{\infty} \frac{1}{\sqrt[3]{x-4}} dx$

3. **(15 points)** Consider the function $f(x) = \begin{cases} 0 \text{ for } x < 4 \\ \dfrac{k}{x^{5/2}} \text{ for } x \geq 4 \end{cases}$ with k a constant.

   (a) **(6 points)** Find a value of k such that $f(x)$ is a probability distribution function.

   (b) **(6 points)** For a random variable X described by the above probability distribution function, find the average value of X.

   (c) **(3 points)** For a random variable X described by the above probability distribution function, find $P(X \leq 9)$.

4. **(10 points)** Consider the curve $y = e^x + 4$ between the points $(0, 4)$ and $(2, 4 + e^2)$.

   (a) **(4 points)** Construct, but do not evaluate, an integral representing the length of this curve.

   (b) **(3 points)** Construct, but do not evaluate, an integral representing the surface area of the surface produced by rotating this curve around the vertical line $x = -3$.

   (c) **(3 points)** Construct, but do not evaluate, an integral representing the surface area of the surface produced by rotating this curve around the x-axis.

5. **(15 points)** Perform the approximations shown below.

   (a) **(5 points)** Using Simpson's rule with $n = 6$, approximate $\int_1^4 \frac{1}{x} dx$. You need not arithmetically simplify your result.

   (b) **(5 points)** Using the trapezoidal rule with $n = 6$, approximate $\int_1^4 \frac{1}{x} dx$. You need not arithmetically simplify your result.

   (c) **(5 points)** Using the midpoint rule with $n = 6$, approximate $\int_1^4 \frac{1}{x} dx$. You need not arithmetically simplify your result.

6. **(15 points)** Consider the curve given by the parametric equations $x = t^2$ and $y = t - t^2$.

   (a) **(8 points)** Find the area under the curve between $t = 0$ and $t = 1$. Your answer need not be arithmetically simplified.

   (b) **(7 points)** Construct, but do not evaluate, an integral representing the arclength of the curve between $t = 0$ and $t = 1$.