

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(25 points)** Identify each of the following series as conditionally convergent, absolutely convergent, or divergent:

(a) **(7 points)** $\sum_{n=1}^{\infty} \frac{n^2}{n!}$.

(b) **(6 points)** $\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^n$.

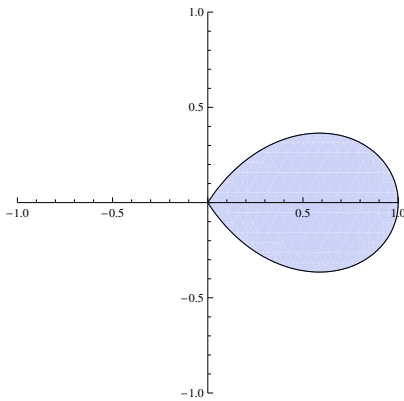
(c) **(6 points)** $\sum_{n=1}^{\infty} \frac{3^n}{n}$.

(d) **(6 points)** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$.

1	/25
2	/15
3	/15
4	/15
5	/15
6	/15
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2. **(15 points)** Consider the curve given by the polar equation $r = 1 - \theta^2$ between $\theta = -1$ and $\theta = 1$ (shown below):

(a) **(9 points)** What is the area of the shaded region? You need not arithmetically simplify your answer.



(b) **(6 points)** Find the arclength of this curve.

3. **(15 points)** Determine whether each of the following series converges, and justify your claim:

(a) **(5 points)** $\sum_{n=1}^{\infty} \frac{1+e^{-n}}{\sqrt{n}}$.

(b) **(5 points)** $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

(c) **(5 points)** $\frac{1}{1} - \frac{1}{8} + \frac{1}{27} - \frac{1}{64} + \frac{1}{125} - \frac{1}{216} + \cdots$.

4. **(15 points)** Answer the following questions about polar coordinates.

(a) **(7 points)** Express the Cartesian formula $y = x^3$ in terms of polar coordinates, with r expressed as a function of θ .

(b) **(8 points)** Express the polar formula $r = \frac{3\sin\theta}{\cos^2\theta}$ in terms of Cartesian coordinates, with y expressed as a function of x .

5. **(15 points)** Answer the following two questions about the power series $\sum_{n=0}^{\infty} \frac{2n(x-2)^n}{5^n}$.

(a) **(8 points)** Find its interval of convergence.

(b) **(7 points)** Find a function which is equal to it.

6. **(15 points)** Does each of the following converge? If so, what does it converge to and why? If not, why not?

(a) **(5 points)** The sequence $\{1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{5}}, \dots\}$.

(b) **(5 points)** The sequence $\{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \dots\}$.

(c) **(5 points)** The series $1 + \frac{4}{3} + \frac{16}{9} + \frac{64}{27} + \frac{256}{81} + \dots$

7. **(5 point bonus)** For which values of x does $\sum_{n=0}^{\infty} \frac{x^n (n!)}{n^n}$ converge?