Show all your work, and explain why you use the arithmetic operations you use in reaching an answer.

1. **(10 points)** Let us call a number "good" if all of its digits are the same parity (i.e. all odd or all even). So, for instance, 62880 and 31713 are both good, but 61407 is not, since 6, 4, and 0 have different parities from 1 and 7.

   (a) **(5 points)** Determine the number of 3-digit "good" numbers; note that numbers, when conventionally written, do not have a leading 0; e.g. "024" is not a three-digit number.

   (b) **(5 points)** Find a formula in terms of n for the number of n-digit "good" numbers.

2. **(10 points)** A modified deck of cards contains five suits and ten cards (numbered 1–10) in each suit. A "straight" is defined as in poker as a hand in which the five cards are numerically consecutive, and of any suit (ignore, for the purposes of this problem, the poker convention that a hand of this sort could also be a "straight flush" or "royal flush" instead). How many different five-card hands (which are not ordered) are straights?

3. **(10 points)** Let us consider license plates which consist of two consecutive groups of either three letters (from a pool of 26 possible) or three numbers (from a pool of 10 possible); e.g. 134-WRG, EEE-EEK, and 852-584 are all possible license plates. However, to avoid confusion, you forbid either group of 3 to consist entirely of the numbers "0", "1", and "5", or the letters "O", "I", and "S" (so we don't allow 115-RTG, because it might be misread as IIS-RTG). How many license plates are possible according to this scheme (you may leave the answer as an unsimplified arithmetic expression, if you like).

4. **(10 points)** Suppose we build a random four-letter "word" by placing in order a random selection from the 21 consonants, a random selection from the 5 vowels, another random consonant, and then another random vowel (most words we produce this way will be nonsense words like "LEFE"). What is the probability that we make a word in which all four letters are different, like "XOFA"?

Musica est exercitium arithmeticae occultum nescientis se numerare animi.
—Gottfried Leibniz