1. **(10 points)** Let $\mathcal{S}$ be a collection of subsets of $\{1, \ldots, n\}$.
   (a) **(4 points)** Demonstrate that it is possible for $|\mathcal{S}|$ (i.e., the number of *subsets* of $\{1, \ldots, n\}$ contained as *elements* of $\mathcal{S}$) to equal 2^{n-1} without any two elements of $\mathcal{S}$ being disjoint from each other.
   (b) **(6 points)** Prove that if $|\mathcal{S}| \geq 2^{n-1} + 1$, then $\mathcal{S}$ contains two elements which are disjoint from each other.
2. **(10 points)** Suppose that the sequence of numbers a_n is given by the recurrence $a_n = 4a_{n-1} + n$ with initial value $a_1 = 0$. Prove that $a_n = \Theta(4^n)$.
3. **(10 points)** If you are given an unsorted list of n numbers, how long will it take to determine whether there are three numbers x, y, and z in the list such that $xy = z$? Give your answer in big-O notation and explain your reasoning.
4. **(10 points)** Ten people go to a party and are introduced; everyone shakes hands with at least one other person. Prove that there must be at least two partygoers who shook hands with the same number of people. Is this still true if not everyone shakes hands with at least one person?

On two occasions I have been asked — "Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?" In one case a member of the Upper, and in the other a member of the Lower House put this question. I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question. —Charles Babbage

 due October 21, 2015