1. **(10 points)** Let a_n represent the number of ways to distribute n blank balls to three labeled boxes such that the first box contains an even number of balls, the second contains an odd number of balls, and the third contains at least 2 balls.

   (a) Find a closed form for the generating function $f(x) = \sum_{n=0}^{\infty} a_n x^n$.

   (b) Decompose this generating function into partial fractions (a calculator or equation-solving system may help).

   (c) Using your partial fraction decomposition, determine a formula for a_n.

2. **(10 points)** Let a_n be the coefficient on x^n in the power-series expansion of $f(x) = \frac{(1+x^2+x^4)x^2}{(1-x)^3(1-x^3)(1-x^{10})}$ (or, equivalently, you could let $a_n = n!f^{(n)}(0)$, using the Maclaurin series). Describe a combinatorial question to which a_n is the answer (i.e., "there are a_n ways to perform the following process…").

3. **(10 points)** Explore the conjugates of the partitions of n into distinct parts. What property defines these partitions, and what is the generating function for the number of partitions with this property?

4. **(10 points)** Let a_n be the number of n-letter strings consisting of the letters A, B, C, and D with at least one A and an even number of Cs.

   (a) Find a formula for the exponential generating function $\sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$.

   (b) Using the above generating function, find a closed formula for a_n.

A matematikus olyan gép, amely kávéból tételeket gyárt. —Alfréd Rényi