1. **(10 points)** You open an account at a bank that pays 5% interest yearly, and deposit a_0 dollars in it. Every year you withdraw $10 times the number of years you have had the account. For example, if you started with $1000, then in the first year you would earn $50 in interest and withdraw $10, leaving $1040, and in the second year would earn $52 and withdraw $20, leaving $1072, and so forth.

   (a) Find a recurrence for a_n, the balance in the account after n years.

   (b) Solve the recurrence to find a closed form for a_n.

   (c) What is the smallest initial deposit which would guarantee that the account never runs out of money?

2. **(10 points)** Let $a_n = 8a_{n-1} - 16a_{n-2} + 3 \cdot 4^n$ with $a_0 = 3$ and $a_1 = -1$. Find a closed form for a_n.

3. **(10 points)** We are making bracelets with 6 stones in a ring, with three different colors of stone. A bracelet must contain at least one stone of each color. Two bracelets are considered to be identical if one is simply a rotation or a flip of the other. How many different bracelets are possible?

4. **(10 points)** A 4×4 grid of squares is filled in, with each of the 16 squares colored black or white. Two colorings are regarded as identical if one can be converted to each other by performing any combination of flipping, rotating, or swapping the two colors (flipping all the black squares to white and vice versa). How many non-identical colorings are there?

Guided only by their feeling for symmetry, simplicity, and generality, and an indefinable sense of the fitness of things, creative mathematicians now, as in the past, are inspired by the art of mathematics rather than by any prospect of ultimate usefulness.

—Eric Temple Bell