No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(12 points)** Answer the following questions about approximation with Newton's method:

   (a) **(6 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^4 - 3x + 1$. Your answer need not be arithmetically simplified.

   (b) **(6 points)** Choose 3 to be an initial approximation of $\sqrt[4]{50}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop a better rational approximation of $\sqrt[4]{50}$; also give an arithmetic expression (which need not be simplified) for the better approximation arising from a *second* iteration of Newton's method.

2. **(24 points)** You are building a box with a closed top (and a closed bottom, and four lateral faces) whose length must be twice its width, but which can be of any height, and which has an interior volume of 9000 cubic inches. What dimensions minimize the material necessary to build the box?

1	/ 12
2	/ 24
3	/ 23
4	/ 17
5	/ 24
6	/ (6)
Σ	/100

3. **(23 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) $\lim_{x\to 0} \frac{x^3}{2e^x-2-2x-x^2}$.

   (b) $\lim_{x\to 0^+} (\ln x)(\sin x)$.

   (c) $\lim_{t\to 16} \frac{4-\sqrt{t}}{4+\sqrt{t}}$.

   (d) $\lim_{q\to\infty} \frac{\ln\sqrt{q}}{q^2}$.

   (e) $\lim_{s\to\infty} \frac{e^{-s}}{\arctan s-\frac{\pi}{2}}$.

4. **(17 points)** Answer the following questions:

   (a) **(9 points)** Given that $g'(x) = 5x^4 - 3x^2 + 4$ and $g(-1) = 2$, find a formula for $g(x)$.

   (b) **(8 points)** Find the general antiderivative of $f(t) = 5\sqrt[3]{t} + 3e^t - (t+2)\left(3 - \frac{2}{t}\right)$.

5. **(24 points)** Answer the following questions related to the shape of the graph of the function $f(x) = 3x^3 - x^4 + 10$.

   (a) **(4 points)** What are $f(x)$'s long term behaviors as x grows very large and as x grows very negative? Describe each direction in either words or symbols.

   (b) **(6 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.

   (c) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (d) **(8 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

6. **(6 point bonus)** If f is continuous and g and h are differentiable, find a formula for $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt$.