This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(15 points)** Answer the following questions.

   (a) **(5 points)** What is the general antiderivative of $\frac{x^2-1}{x^2+1} - e^x + \csc x(\csc x + \cot x)$?

   (b) **(5 points)** If $f''(x) = 6x$, $f'(2) = 3$, and $f(1) = 0$, find a formula for $f(x)$.

   (c) **(6 points)** Simplify the expression $\frac{d}{dx}\int_{x^2}^{x^3} \ln(t^2 - 7)dt$.

2. **(20 points)** You have 1200 square centimeters of material with which to make a box with a square base and an open top. Find the dimensions which maximize the volume of the box.

1	/ 15
2	/ 20
3	/ 15
4	/ 18
5	/ 16
6	/ 16
7	/ 12
8	/ 15
9	/ 15
10	/ 8
11	/ (6)
Σ	/150

3. **(15 points)** The *lemniscate* is a curve satisfying the equation $2(x^2+y^2)^2 = 25(x^2-y^2)$.

   (a) **(10 points)** Find a formula for $\frac{dy}{dx}$ on this curve in terms of x and y.

   (b) **(5 points)** Find the equation of the tangent line to the lemniscate at the point $(3,1)$.

4. **(18 points)** Evaluate the following integrals:

   (a) **(6 points)** $\int_0^4 x\sqrt{16-x^2}dx$.

   (b) **(6 points)** $\int \frac{x^3}{x^4+1}dx$.

   (c) **(6 points)** $\int_0^{\pi/3} \cos\theta + 2\sec\theta\tan\theta d\theta$.

5. **(16 points)** A ten-foot-long ladder is leaning against a wall, with the base of the ladder six feet from the wall. The base is sliding away from the wall at a rate of half a foot per hour.

   (a) **(8 points)** How quickly is the top of the ladder slipping down the wall?

   (b) **(8 points)** How quickly is the angle between the ladder and the floor changing?

6. **(16 points)** Compute the following expressions:

   (a) **(6 points)** Compute $\frac{d}{dx}\sqrt{\arctan\sqrt{x}}$.

   (b) **(6 points)** Given $f(t) = \tan\frac{e^t}{\arcsin t}$, find $f'(t)$.

   (c) **(4 points)** Find $\int \frac{d}{ds}\frac{s^3}{\sqrt{s^2+5}}ds$.

7. **(12 points)** Consider the function $g(x) = \frac{x}{x^2+9}$.

   (a) **(5 points)** Identify the zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which, and if any features are not present, say so.

   (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.

   (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

8. **(15 points)** Determine the following limits.

   (a) **(5 points)** Evaluate $\lim\limits_{\theta \to 0} \frac{\theta - \sin\theta}{\theta - \tan\theta}$ or demonstrate that it cannot be evaluated.

   (b) **(5 points)** Evaluate $\lim\limits_{t \to 0} \frac{e^t - 1 - t}{t^2}$ or demonstrate that it cannot be evaluated.

   (c) **(5 points)** *Using the difference quotient*, find the derivative with respect to x of the function $f(x) = 20 + 3x - 5x^2$. You may not use L'Hôpital's rule for this problem.

9. **(15 points)** Let $f(x) = 2 + 2x^2 - x^4$.

   (a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing? Label which is which.

   (b) **(3 points)** What are the critical points of $f(x)$? Is each a local maximum, a local minimum, or neither?

   (c) **(7 points)** Determine where $f(x)$ is concave up and where it is concave down, and identify points of inflection.

10. **(8 points)** Answer the following questions about the function $f(x) = \frac{x^{2/3}}{x+1}$.

    (a) **(4 points)** What is the domain of $f(x)$?

    (b) **(4 points)** Where does the derivative of $f(x)$ exist?