1. **(4 points)** *Determine a value of k such that the function* $f(x) = \begin{cases} kx^2 \text{ if } x \leq 3 \\ k - 5x \text{ if } x > 3 \end{cases}$ *is continuous everywhere.*

   The two pieces of this function are each continuous over their entire domain, so the sole concern is to make sure that $f(x)$ is continuous at the transition point; namely, at $x = 3$. Thus we want $\lim_{x\to 3^-} f(x)$, $\lim_{x\to 3^+} f(x)$, and $f(3)$ to all have the same value. These can be evaluated to be $k \cdot 3^2$, $k - 5 \cdot 3$, and $k \cdot 3^2$ respectively, so what we require is simply that $k \cdot 3^2 = k - 5 \cdot 3$; this algebraic equation is easily solved to find that $k = \frac{-15}{8}$.

2. **(5 points)** *Prove, using the epsilon-delta definition of a limit, that* $\lim_{x\to -1} -3x + 4 = 7$.

   The statement $\lim_{x\to -1} -3x + 4 = 7$ is an assertion that, for every value $\epsilon > 0$, a value δ can be furnished such that, if $0 < |x - (-1)| < \delta$, then $|-3x + 4 - 7| < \epsilon$. We may justify this assertion by explicity determining how δ is calculated from ϵ to make this inference true.

$$\begin{aligned} |-3x + 4 - 7| &< \epsilon \\ |-3x - 3| &< \epsilon \\ |-3(x+1)| &< \epsilon \\ |-3| \cdot |x+1| &< \epsilon \\ 3|x - (-1)| &< \epsilon \\ |x - (-1)| &< \frac{\epsilon}{3} \end{aligned}$$

   so we may declare that the choice of δ equal to $\frac{\epsilon}{3}$ is sufficient to meet whatever challenge we are given.

3. **(6 points)** *Evaluate each of the following infinite limits or demonstrate that the limit does not exist.*

   (a) $\lim_{r\to+\infty} \frac{3r-5r^3}{100r^2-2r+1}$
   Dividing both numerator and denominator by r^2 to make the denominator finite, we find that

$$\lim_{r\to+\infty} \frac{3r - 5r^3}{100r^2 - 2r + 1} = \lim_{r\to+\infty} \frac{\frac{3r-5r^3}{r^2}}{\frac{100r^2-2r+1}{r^2}} = \lim_{r\to+\infty} \frac{\frac{3}{r} - 5r}{100 - \frac{2}{r} + \frac{1}{r^2}}$$

   and in this expression, the denominator will tend towards $100 - 0 + 0$, while the numerator tends towards $0 - 5r$, so as r gets very large, this expression is approximately $\frac{-r}{20}$, whose limit does not exist — specifically, it decreases without bound, although it is not necessary to describe it in such detail.

   (b) $\lim_{x\to-\infty} \frac{x^2}{2-18x^5}$
   Dividing both numerator and denominator by x^5 to make the denominator finite, we find that

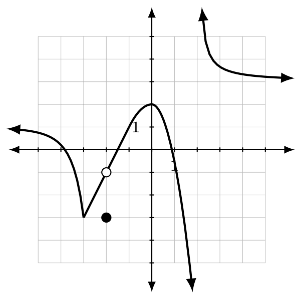
$$\lim_{x\to-\infty} \frac{x^2}{2 - 18x^5} = \lim_{x\to-\infty} \frac{\frac{x^2}{x^5}}{\frac{2-18x^5}{x^5}} = \lim_{x\to-\infty} \frac{\frac{1}{x^3}}{\frac{2}{x^5} - 18} = \frac{0}{0 - 18} = 0.$$

   (c) $\lim_{u\to-\infty} \frac{u^4-7u^2+3u}{6u^4+2u-1}$

Dividing both numerator and denominator by u^4 to make the denominator finite, we find that

$$\lim_{u\to-\infty} \frac{u^4-7u^2+3u}{6u^4+2u-1} = \lim_{u\to-\infty} \frac{\frac{u^4-7u^2+3u}{u^4}}{\frac{6u^4+2u-1}{u^4}} = \lim_{u\to-\infty} \frac{1-\frac{7}{u^2}+\frac{3}{u^3}}{6+\frac{2}{u^3}-\frac{1}{u^4}} = \frac{1-0+0}{6+0-0} = \frac{1}{6}.$$

4. **(5 points)** *Below is the graph of a function $f(x)$. Answer the questions asked.*



What is $\lim_{x\to-\infty} f(x)$*?*

What is $\lim_{x\to+\infty} f(x)$*?*

At what values of x is the function $f(x)$ discontinuous?

Because the graph of the function trails off on the left towards the height $y = 1$, we would say that $\lim_{x\to-\infty} f(x) = 1$; likewise on the right, the trailing off towards a height of 3 is denoted by the statement that $\lim_{x\to-\infty} f(x) = 3$.

The graph of $f(x)$ possesses discontinuities at places where either the limit does not exist, the function value does not exist, or the two do not agree. At $x = -2$, there is a discontinuity because $\lim_{x\to-2} f(x) = -1$ while $f(-2) = -3$. There is also a discontinuity at $x = 2$ because neither the limit nor the function value exists there.

5. **(2 point bonus)** *Prove (formally) that the equation $x = \tan x$ has infinitely many solutions.*